INFLUENCE OF WHEEL/RAIL CONTACT GEOMETRY ON THE BEHAVIOUR OF A RAILWAY VEHICLE AT STABILITY LIMIT

Oldrich Polach
Bombardier Transportation
Winterthur
Switzerland
oldrich.polach@ch.transport.bombardier.com

Abstract
The paper presents several possible methods for nonlinear stability analysis. Irrespective of the method applied it can be demonstrated that, at the stability limit, either a limit cycle with a large amplitude manifests itself abruptly, or a limit cycle with very small amplitude develops first, whereby the amplitude then only slowly accumulates with increasing speed. This phenomenon is analysed in conjunction to the quasi-linear wheel/rail contact. The quasi-linearisation of the wheel/rail contact geometry indicates the behaviour to be anticipated at the stability limit, thereby enabling a better understanding of the nonlinear calculations and measuring results.

Key words
Rail vehicle dynamics, stability, wheel/rail contact.

1 Introduction

Stability assessment plays an important role in railway vehicle dynamics. Depending on which body is predominantly excited by the oscillation form, differentiation between the car body instability and bogie instability can be made. The bogies demonstrate limit cycles for all speeds higher than the critical speed, whereas car body instability can sometime be suppressed with increasing speed. Bogie instability possesses safety relevance, as the evolving high lateral forces between wheel and rail could cause track displacement which in turn could lead to derailment (Fig. 1). With this in mind, bogie stability assessment plays a significant role during railway vehicle engineering.

During 1960/70 a theoretical comprehension of railway vehicle stability came into being as a result of studies founded on the linearised models; see the monographs of *Wickens* [Wickens, 2003] and the paper from *Knothe and Böhm* [Knothe and Böhm, 1999] for historical overview and further references. At a later date the nonlinearities of wheel/rail combination were also taken into consideration, see the paper from *True* [True, 1999].

Due to the wide range of input conditions and methods used for the stability assessment, the stability analysis provides probably the most diversified type of running dynamics calculations. Methods such as nonlinear and linearised calculations can be applied in various versions and the results can vary dependent on the method used.

Fig. 1 Track shift after a test run with an older diesel engine (from [Köhler et al., 2003])

The linearised and nonlinear methods demonstrate significant differences and are not always comparable. The nonlinear analyses using three-dimensional vehicle models build up in a modern multi-body simulation tool, as it is the state-of-the-art in the railway vehicle industry today, allow detailed nonlinear stability analysis for the specified conditions. However, because of the variety and an important influence of the nonlinear wheel/rail contact geometry, the stability assessment results can lead to large differences dependent on the method and conditions used.

This paper analyses the contact geometry wheelset/track and its influence on the behaviour of a bogie at the stability limit. Different methods of nonlinear stability analysis as they can, or may be used in industrial applications are introduced and compared. The linearisation used to calculate the quasi-linear equivalent conicity is presented. Further, the vehicle behaviour under the presence of different contact geometries leading to different linearised functions is compared on selected examples. It is shown, that the quasi-linearisation of the wheel/rail contact geometry suggests the behaviour to be anticipated at the stability limit.

2 Nonlinear stability analysis

2.1 Overview

The stability behaviour is highly sensitive to the nonlinearities of wheel/rail contact. These nonlinearities are triggered by the creep forces, and primarily by the contact geometry between wheelset and track.

There are several possible criteria for the classification of the nonlinear methods for the bogie stability assessment.

One possible classification is according to the analysed values. It can be

- wheelset displacement (lateral or yaw displacement)
- lateral forces between wheelset and track (sum of guiding forces, called also track shifting force)
- lateral acceleration on the bogie frame.

Further criterion for the classification can be the definition of the stability limit. From a mechanical viewpoint, a system possessing the capability to oscillate can be viewed as stable if the oscillations decrease following discontinuation of the excitation. Should a limit cycle having constant amplitude arise at a particular running speed, this speed is defined as a critical speed. However, in railway practice and in the specifications concerning the vehicle acceptance as described in the code of International Union of Railways [UIC Code 518, 2003], in the draft of European standard [prEN 14 363, 2002] and in the US standard 49CFR238 [49CFR238, FRA, 2003], the bogie stability is defined by way of the limit values of the measuring quantities. Should the limit value be exceeded, the running behaviour is described as being unstable.

Another classification criterion is the type of excitation applied. Differentiation can be made between computer simulations

- without excitation - running on ideal track, starting from the limit cycle and reducing the speed until a stable bogie motion is achieved
- with excitation by a singular irregularity, followed by an ideal track (or with short irregularity sequence followed by an ideal track), with or without variation of the excitation amplitude
- with excitation by stochastic (measured) track irregularity as used during the acceptance test of the vehicles.

In the following, the different methods of nonlinear stability analysis as they can, or may be used in industrial applications are introduced and compared on two selected examples of contact geometry wheelset/track with high equivalent conicity. Comparing the equivalent conicity for the lateral amplitude of 3 mm, which is used to characterise the contact geometry in railway practise, both examples demonstrate nearby the same value of approx. 0.4.

The simulations are carried out in the simulation tool Simpack with a completely nonlinear model of a four-car articulated motor unit, see Fig. 2. The friction between wheel and rail was set to 0.4 (dry rail). The results are given for the trailing wheelset of the first bogie, at which the stability limits are first reached. In the following, the methods are presented classified according to the excitation applied.

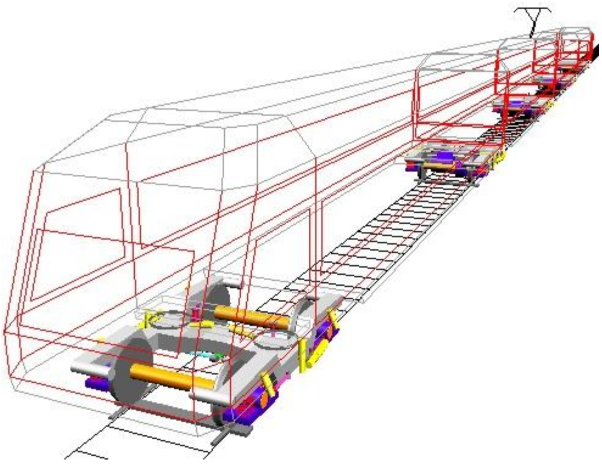

Fig. 2 Simulation model of a four-car articulated vehicle

2.2 Method without excitation

In this case a high speed during which the bogie moves in a limit cycle is used as initial condition and a continuous speed reduction takes place as applied e.g. in the investigation concerning the tuning of freight wagon bogies using inter-axle linkages [Orlova et al., 2002]. The speed at which the vibrations subside is designated as being the critical speed, see Fig. 3. For the contact geometry 04A the vibrations stop abruptly, whereas for the contact geometry 04B the wheelsets continue to vibrate in a small limit cycle, only stabilising at a significantly lower speed, which subsequently leads to significantly differing critical speeds at the same equivalent conicity. Both simulations start with flange-to-flange limit cycle of the wheelset, but the behaviour of the wheelsets and bogie during the speed reduction is different as shown also in the Fig. 4, where the phase diagram of the lateral displacement of the leading and trailing wheelset is presented.

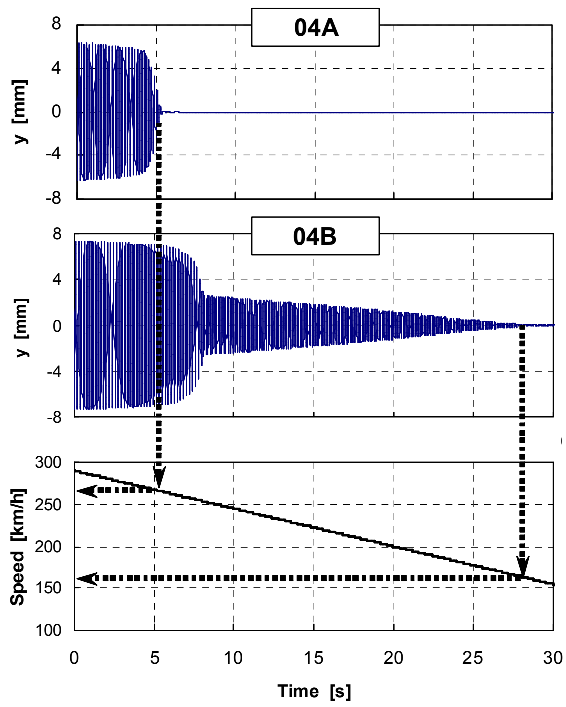

Fig. 3 Simulations of run with decreasing speed

2.3 Methods with single excitation

Investigating damping behaviour following a single lateral track excitation, stability can be assessed; however the damping behaviour can differ for various contact geometries as can be seen in Fig. 5. Furthermore, because of the geometrical nonlinearities, multiple solutions can exist. The existence of multiple solutions in nonlinear dynamical parameter dependent problems is related to a phenomenon called "bifurcation". The usual way to present this phenomenon is bifurcation diagram [True, 1999], [Schupp, 2004].

When analysing the stability of railway vehicles, the bifurcation diagram displays amplitude of the limit cycle in function of speed as demonstrated on Fig. 6. Two typical situations can result: subcritical or supercritical bifurcation, see Fig. 7 [True and Kaas-Petersen, 1983]. In case of subcritical bifurcation there is a speed range at which the solution can "jump" between a stable damped movement and a limit cycle depending on the excitation amplitude. In accordance with the investigated profile combinations, the bifurcation diagrams assume two basically different forms, see Fig. 8. In the case 04A it is a subcritical bifurcation with an unstable attractor and multiple solutions, whereas in the case 04B the solution corresponds to a supercritical bifurcation diagram, where the amplitude of the limit cycles increases continuously.

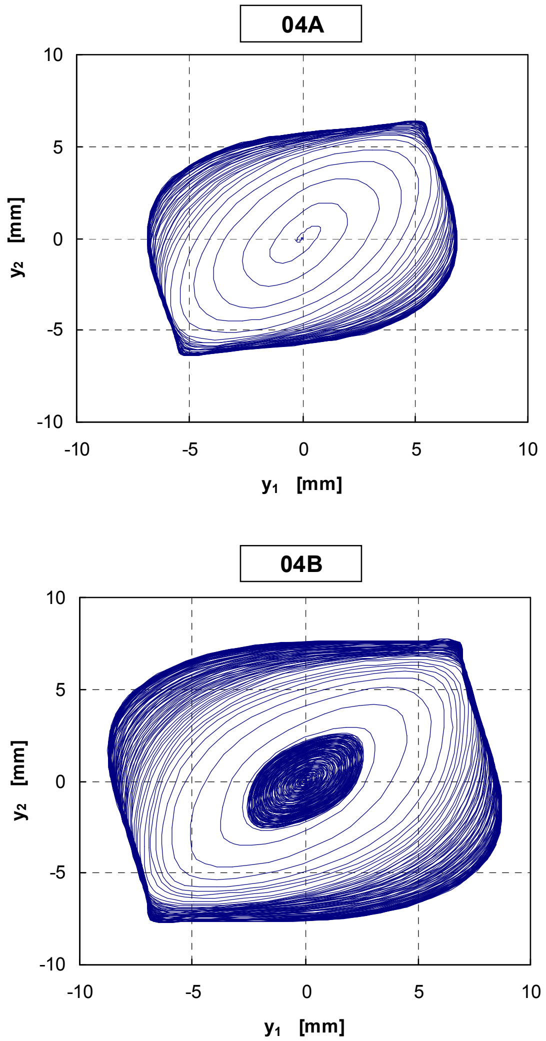

Fig. 4 Diagram of lateral displacements y_1 and y_2 of leading and trailing wheelset, respectively.

2.4 Methods with stochastic excitation

To assess the bogie stability during the rail vehicle engineering, also the methods specified for measurements and acceptance tests can be applied. Running on straight track with stochastic (usually measured) irregularities is simulated and instability criteria for vehicle acceptance tests are applied for assessment.

In the presented comparison, the critical speeds evaluated using measuring criteria according to [UIC Code 518, 2003] and [prEN 14 363, 2002] were applied:

- lateral forces between wheelset and track (sum of guiding forces)
- lateral acceleration on the bogie frame

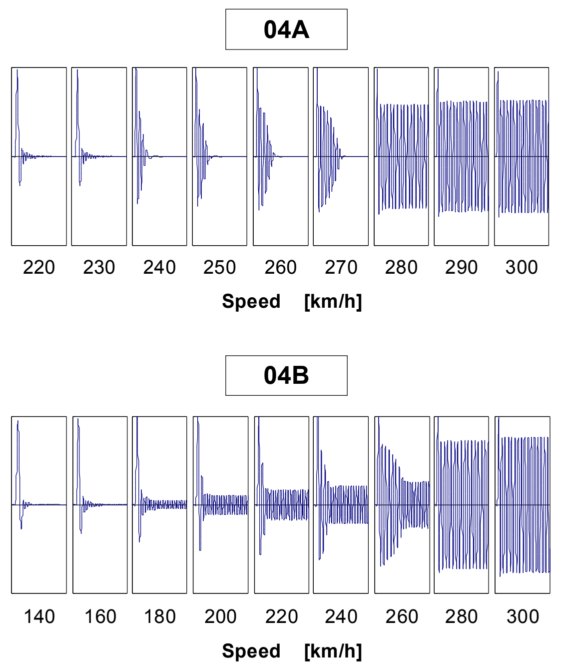

Fig. 5 Simulations of lateral wheelset displacement following a single lateral excitation

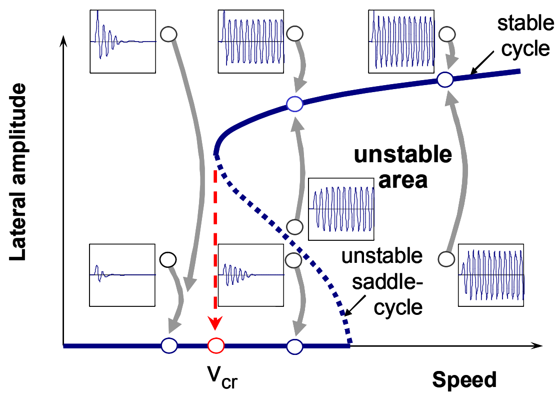

Fig. 6 Bifurcation diagram as result of the simulations of wheelset behaviour after an excitation

As critical speed, the speed was referenced at which the bogie instability limit is just achieved. A comparison of the results normalized with the limit value can be seen in Fig. 9. Although the calculated critical speeds are more similar than applying other methods, the progression of the investigated criteria with increasing speed is different.

Detailed description of presented stability analysis applying computer simulation of run on measured irregularities can be found in [Polach and Vetter, 2004].

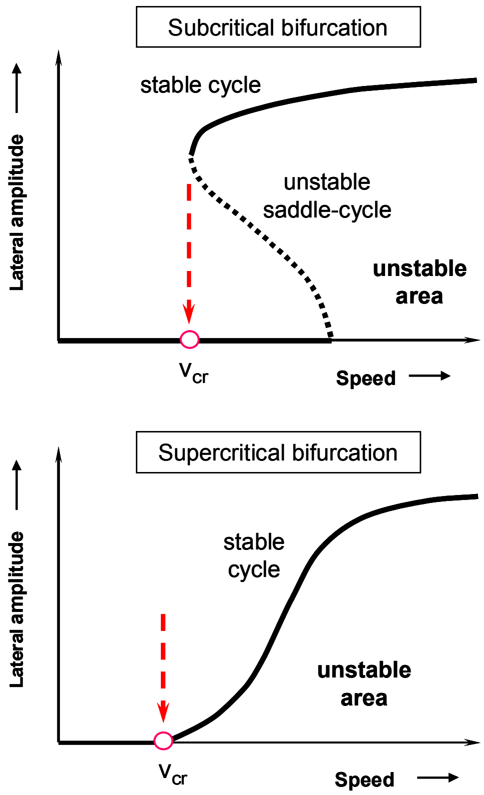

Fig. 7 Types of bifurcation diagrams

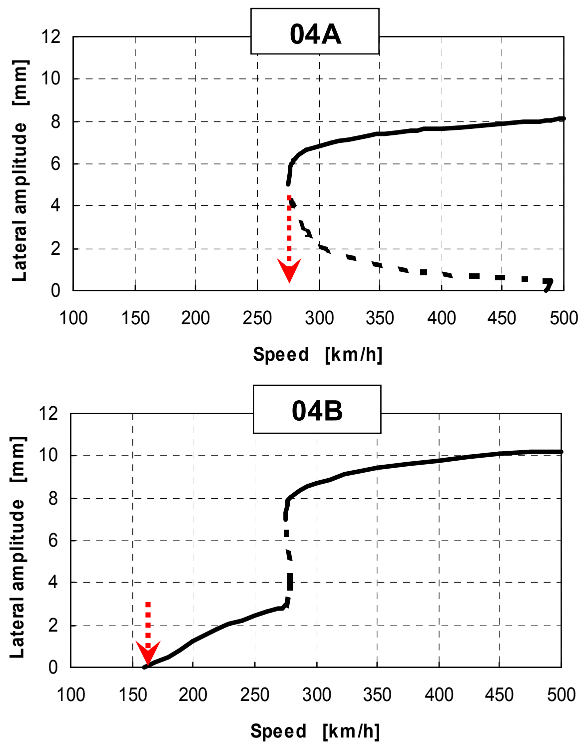

Fig. 8 Bifurcation diagrams for investigated profile combinations

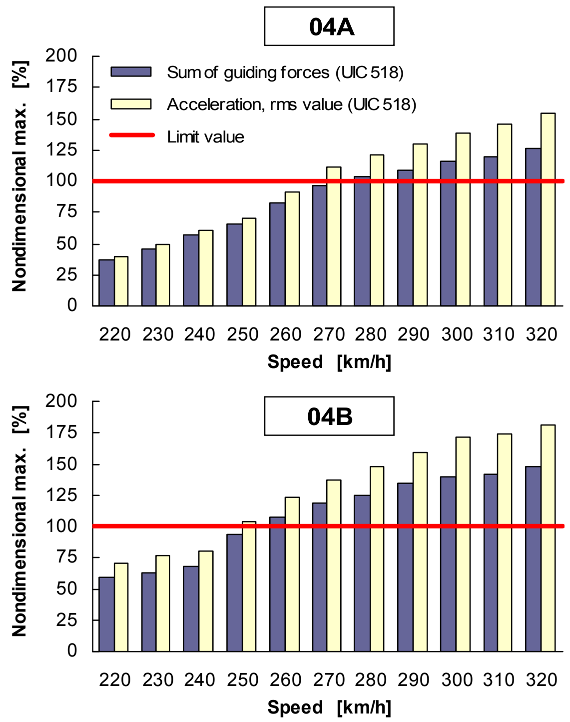

Fig. 9 Normalised results of stability analysis running on track with irregularities

2.5 Discussion

As can be seen from the presented comparisons, the behaviour of the wheelsets and bogie at the stability limit can be significantly different even for the same equivalent conicity. The differences observed make the assessment of the risk of instability more difficult. They are caused mainly by the nonlinearity of the contact geometry wheelset/track.

In the following, the quasi-linearisation of the contact geometry wheelset/track will be presented. The differing behaviour at the stability limit will be analysed in conjunction to this linearised parameters.

3 Linearisation of the contact geometry wheelset/track

In railway applications, quasi-linearisation is largely resorted to in order to characterise the contact geometry wheelset/track with one parameter only – with the so-called equivalent conicity. To identify the equivalent conicity, characteristics of the wheelset/track pairing is "replaced" with an "equivalent wheelset" with conical wheel tread surface, whereby this "replacement" only possesses validity for one value of the wheelset lateral amplitude. The equivalent conicity is then the conicity of a conical wheelset which, at the prescribed lateral amplitude, demonstrates similar wavelike motion as the examined wheelset.

If the wheelset with conical tread profiles moves laterally with a displacement y from its centred position, the rolling radii of the right wheel r_r and left wheel r_l are different. The conicity λ of the wheel tread can be expressed as function of wheelset rolling radii difference Δr

$$\lambda = \frac{r_r - r_l}{2y} = \frac{\Delta r}{2y} \qquad \textbf{(1)}$$

The "describing function" y_{DF} of rolling radii difference Δr in function of lateral wheelset displacement y is used when evaluating the equivalent conicity of the pairings of wheel and rail profiles. There exist several methods for determining the equivalent conicity which may partially lead to differing results for the same conditions. In the following analysis, method of harmonic linearisation [Mauer, 1991] will be applied.

To linearise the describing function $y_{DF} = f(x)$ we should minimise the quadratic error between the nonlinear function $y_{DF} = f(x)$ and the quasi-linear approach $y_{DF} = k \cdot x$

$$\Delta^2 = \left(f(x) - k \cdot x\right)^2 \qquad \textbf{(2)}$$

This mean that there is an extremum and therefore

$$\frac{\partial}{\partial k}\left(\Delta^2\right) = 0 \qquad \textbf{(3)}$$

After a differentiation we get the coefficient k as

$$k = \frac{x \cdot f(x)}{x^2} \qquad \textbf{(4)}$$

With the harmonic approach with an amplitude A

$$x(\omega t) = A \cdot \sin \omega t \qquad \textbf{(5)}$$

we get the "describing function" of the linear factor $k(A)$ which is dependent on the amplitude A of the harmonic linearisation

$$k(A) = \frac{1}{\pi \cdot A} \int_0^{2\pi} f\left(A \cdot \sin\varphi\right) \cdot \sin\varphi \, d\varphi \qquad \textbf{(6)}$$

Setting the difference of wheelset rolling radii $\Delta r = f(y)$ as the "describing function" we get the equivalent conicity as nonlinear function of the linearisation amplitude A

$$\lambda(A) = \frac{1}{2\pi A} \int_0^{2\pi} \Delta r\left(A \sin\varphi\right) \cdot \sin\varphi \, d\varphi \qquad \textbf{(7)}$$

Based on the Equation (7), the equivalent conicity can be obtained by numerical integration of the nonlinear geometrical function of wheelset rolling radii difference Δr.

As the contact points between wheel and rail usually do not move continuously in function of wheelset lateral displacement and large “jumps” between the contact points are not rare, the function of rolling radii difference and consequently also the equivalent conicity are not monotonous functions. The relation between the equivalent conicity function and bogie’s behaviour at the stability limit is discussed in the next chapter.

4 Linearised contact parameters and bogie’s behaviour at the stability limit

The linearisation of the contact geometry wheelset/track and the equivalent conicity is used not only for linear calculations but also to characterise the track (combining the measured rail profiles with theoretical wheel profiles), the geometry of worn wheel profiles (combining the measured wheel profiles with theoretical rail profiles), or the geometrical conditions of the wheelset on track (combining theoretical or measured wheel and rail profiles). For the analysis of wheelset’s and bogie’s behaviour, the contact conditions are relevant occurring for wheelset amplitude up to approximately a half of the gauge clearance. The value of equivalent conicity for a wheelset lateral amplitude of 3 mm is used to characterise the contact geometry wheelset/track in railway praxis [UIC Code 518, 2003].

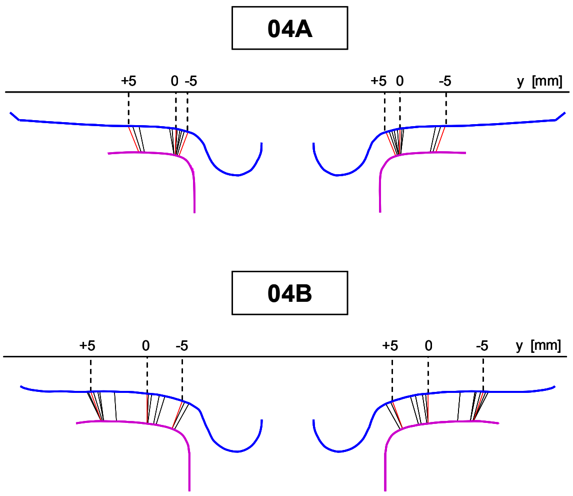

Fig. 10 The contact geometry of the investigated profile pairings

The analysis of the contact geometry wheelset/track as used for quasi-linearisation can also enable a better judgement of the simulation results. Irrespective of the method applied, it was demonstrated that, at the stability limit, either a limit cycle with large amplitude manifests itself abruptly, or a limit cycle with very small amplitude develops first, whereby the amplitude then only slowly increases with increasing speed. Let us compare the describing functions of rolling radii difference and the equivalent conicity function of the contact geometry examples analysed in Chapter 2. The positions of the contact points on wheel and rail at different lateral wheelset displacements are visualised in Fig. 10. Fig. 11 shows the rolling radii difference functions. For the pairing 04A, there is nearby no movement of the contact point below 2 mm wheelset displacement, but there is large movement of the contact point on the left rail for amplitudes between 2 and 3 mm, which leads to a step in the function of rolling radii difference. The function is progressive between 0 and 3 mm. For the pairing 04B, there is the largest movement of the contact point on the left rail for the wheelset displacement between 0 and 1 mm, leading to a degressive form of the rolling radii difference function.

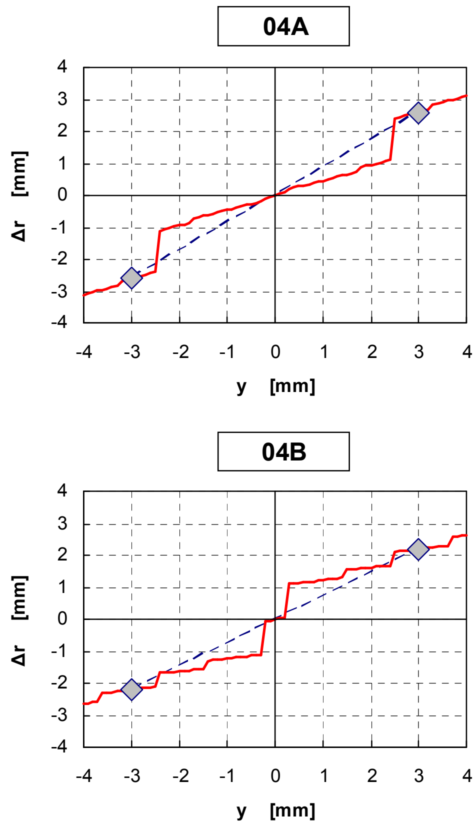

Fig. 11 Rolling radii difference Δr in function of lateral wheelset displacement y

As the rolling radius difference can show large variation, also the equivalent conicity can achieve various progressions in the function of amplitude. Fig. 12 presents the corresponding equivalent conicity functions calculated for rigid contact as usually used in railway praxis, and for quasi-elastic contact as implemented in SIMPACK [Netter et al., 1998]. The quasi-elastic contact considers the contact elasticity and demonstrates more realistic contact conditions than the rigid contact. It is therefore applied in presented simulations. In spite of small differences between the rigid and quasi-elastic contact, the diagrams show similar tendencies. Whereas the conicity for large amplitudes (above 6 mm) achieves high value in both cases, the progression for smaller amplitudes demonstrates either an increasing or decreasing form in accordance with the rolling radius difference presented in Fig. 11. The contact geometry 04A indicates an increase of conicity in the lateral amplitude range below 3 mm. Conversely, the contact geometry 04B indicates a decreasing conicity in the lateral amplitude range of 0 to 3 mm.

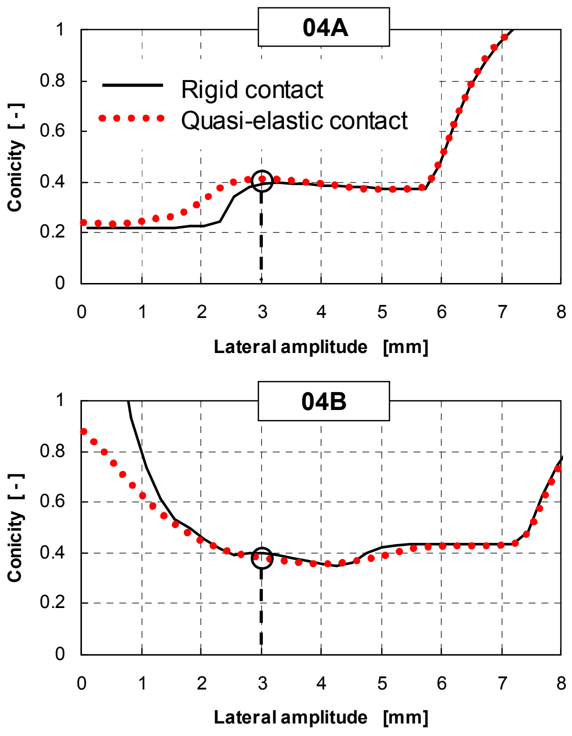

Fig. 12 Equivalent conicity diagrams

Comparing the equivalent conicity function with the results of the stability analysis presented in Chapter 2, a relation between the equivalent conicity and the behaviour at the stability limit can be observed. An increasing equivalent conicity function in the range of wheelset lateral amplitude below approx. 3 mm leads to an abrupt limit cycle with large amplitude (subcritical bifurcation), whereas a decreasing conicity for wheelset lateral amplitude below 3 mm leads to supercritical bifurcation with limit cycles with small amplitudes only slowly accumulating with increasing speed. This relation was also observed for other wheelset/track contact geometries and confirmed to be a regularity enabling a better judgement of the simulation results at the stability limit.

The equivalent conicity for specified linearisation amplitude characterises the wheelset behaviour during the wavelike motion with this amplitude. Should the linearisation for the individual amplitudes be carried out, and the quasi-linear critical speed for those linearisation parameters calculated, we achieve a set of values with a distribution similar to those demonstrated in the bifurcation diagram, see Fig. 13. This figure demonstrates that the quasi-linear solution is usually similar to the nonlinear analysis if the quasi-linear parameters calculated for the specified wheelset amplitude are applied. The differences between nonlinear and linearised calculations as referenced e.g. in [True, 1999], [Knothe and Böhm, 1999] occur mainly if the linearisation is carried out around the centred position, for amplitude → 0, which is usually not used in railway applications.

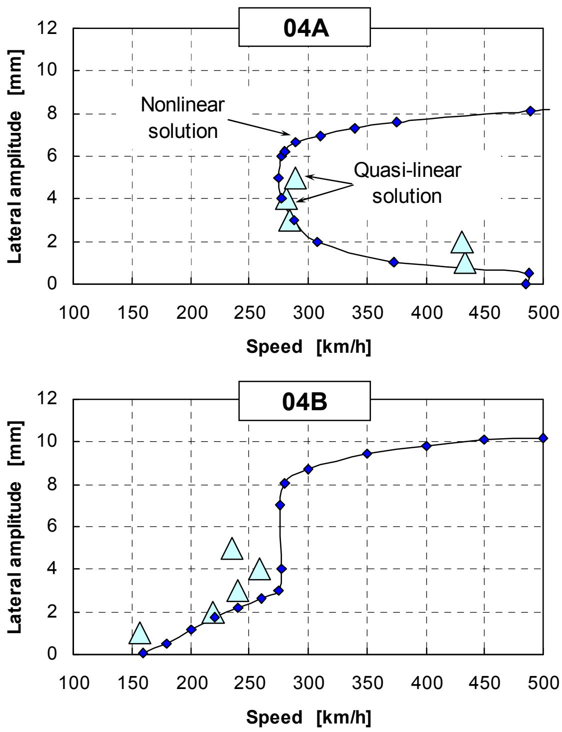

Fig. 13 Comparison of nonlinear and quasi-linear solutions of examined combinations wheelset/track

The behaviour of the wheelsets and bogie at the stability limit, e.g. the transition from the stable run to a limit cycle, the occurrence of a limit cycle and the amplitude of a limit cycle, are very sensitive to the contact geometry wheelset/track. The behaviour demonstrated in computer simulations can vary dependent on the modelling of the contact geometry. Small differences in the contact geometry caused e.g. by different input data exactness or by filtering of the profile data may lead to differences in the simulated behaviour. The results can differ also in dependence of the application of rigid or elastic wheel/rail contact. The analysis of the contact geometry wheelset/track as used for the quasi-linearisation implies the nonlinear behaviour at the stability limit and can support the understanding of the nonlinear analyses.

5 Conclusion

The analysis of the contact geometry wheelset/track may not only serve as input for the linearised calculation, but also imply the nonlinear behaviour at the stability limit, thereby enabling better understanding of the nonlinear calculations and measuring results. A decreasing equivalent conicity function in the range of amplitudes below approx. 3 mm leads to a supercritical bifurcation and a limit cycle with small amplitude only slowly accumulating with increasing speed, whereas the increasing equivalent conicity is linked with subcritical bifurcation characterised by an abrupt transition from stable behaviour to a pronounced limit cycle with large amplitude.

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