# Comparability of the non-linear and linearized stability assessment during railway vehicle design

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## INTRODUCTION

A diversity of feasible methods exists with which computer analyses of running stability can be carried out during railway vehicle design. On the one hand these consist of linearized analyses of the eigenvalues, on the other of non-linear simulations which can also be realised with varying types of excitation and also assessed according to differing criteria. The question of the applicability of linearized calculations during the railway vehicle design is very topical, as was confirmed during the discussion at the 18<sup>th</sup> IAVSD Symposium in Kanagawa 2003 [1].

The non-linear studies of rail vehicle stability presented in several publications usually state, that the linearized calculation leads to higher critical speed than the non-linear analysis and demonstrate this conclusion by a typical form of the bifurcation diagram of a railway vehicle [2, 3], see Fig. 1. The paper compares linearized and non-linear methods of stability assessment as they can, or may be used in industrial applications, on various examples of contact geometry wheelset/track with high equivalent conicity.

#### 1. LINEARIZED CALCULATIONS

It is important to state that the linearized analyses applied during railway vehicle design, as discussed in this paper, do not accord with the calculation of critical speed  $v_{lin}$  in Fig. 1. Instead, a method of quasi-linearization is applied, with which the linearized wheel/rail parameters are calculated for the specified amplitude of the wheelset lateral movement. In the paper the term 'linearized calculation' always signifies a calculation with the application of quasi-linear wheel/rail contact.

A value of equivalent conicity used to specify the wheel/rail contact geometry in linear calculations is influenced both on the side of the rail by the rail profile, the rail inclination and the track gauge, as well as on the side of the wheelset, by the wheel profile, the back-to-back wheel distance and the diameter of the leftand right-hand wheels. In addition to the equivalent conicity, the linearized wheelset/track model depends upon the contact angle parameter and the roll parameter [4]. In the linear stability analyses, these parameters are described as functions of the conicity. During linearization of the effective wheelset/track pairings these values often deviate from the given functions and achieve differing values at the same conicity. For example, at a conicity of 0.4 the roll parameter can be located between 0.04 and 0.12. Consequently, the critical speed of the examined vehicle demonstrates a distribution of almost 40 km/h, see Fig. 2.

As the linearized methods represent a simplification, the uncertainty of the results must be secured by a safety margin. Therefore, a minimum of 5% of critical damping is usually requested to ensure stable running. In order to represent dry and clean conditions in the wheel/rail contact, no reduction of Kalker's factor is usually applied in linearized stability assessment.

# 2. NON-LINEAR CALCULATIONS

There are several non-linear calculation methods, which lead to different results in dependence on the kind of stability analysis and criteria applied, see [5]. The methods of non-linear stability analysis can be classified

# XIX IAVSD Symp., Paper #42

according to analysed values or type of excitation applied. Further criterion for the classification can be the definition of the stability limit. From a mechanical viewpoint, a system possessing the capability to oscillate can be viewed as stable if the oscillations decrease following discontinuation of the excitation. Should a limit cycle having constant amplitude arise at a particular running speed, this speed is defined as a critical speed. However, in railway practice and in the specifications concerning the vehicle acceptance [6] the bogie stability is defined by way of the limit values of the measuring quantities. Should the limit value be exceeded, the running behaviour is described as being unstable.



Fig. 1. Bifurcation diagram with comparison of linear and non-linear critical speed

Fig. 2. Influence of contact angle parameter and roll parameter on the critical speed for the conicity of 0.4

To represent dry wheel/rail conditions, friction coefficient between 0.4 - 0.5 is applied. The contact geometry wheelset/track is described with the effective profiles. The fact that one equivalent conicity value can be represented by varying profile pairings leads to deviations in the results. Furthermore, the resultant critical speed is influenced by the method of analysis, the type of excitation and the choice of criterion, see [5]. This leads to an extensive dispersion of the results, as can be seen exemplary on Fig. 3 for two contact geometries 04A and 04B which both exhibit the same equivalent conicity of approx. 0.4. Depending on the methods and criteria the differences are even larger than with the linearized calculations as can be seen comparing Fig. 2 and Fig. 3 which present analyses of the same vehicle and the same equivalent conicity. The differences depend mainly on the shape of the wheel/rail contact geometry as demonstrated in [5]. The greatest deviations of the resultant critical speeds take place in case of supercritical bifurcation when small limit cycles occur and these are taken into account for the stability assessment. In other cases the resultant critical speeds achieve similar values for all non-linear methods tested.



Fig. 3. Critical speed identified applying differing non-linear methods and stability criteria for two different wheel/rail combinations 04A and 04B with the same equivalent conicity of 0.4

For the presented comparison of non-linear and linearized calculations, the non-linear wheel/rail contact geometry was applied which leads to supercritical bifurcation. Two mostly used methods in rail vehicle engineering were applied:

- damping behaviour behind a single lateral excitation
- run on measured track irregularities and analysis of the sum of guiding forces according to [6].

## 3. COMPARISON OF RESULTS AND CONCLUSION

The non-linear and quasi-linear calculation methods were compared for:

- 2 vehicle types (a four-car articulated vehicle with Jakobs' bogies and yaw dampers, a conventional four-axle passenger coach without yaw dampers)
- for linear analyses with variation of the Kalker's factor (0.67 and 1.0) and variation of the minimum damping considered when evaluating the critical speed (0% and 5% of critical damping)
- for non-linear analyses with variation of the wheel/rail friction coefficient (0.4 and 0.5) and the wheel/rail pairings to set up the specified conicity (on the one hand by altering the track gauge, on the other by wearing of the rail profile).

The comparisons demonstrated that linearized calculations under conditions applied during railway vehicle engineering (minimum damping 5%, Kalker's factor 1.0) deliver a lower critical speed than the non-linear analyses for all investigated examples and are therefore on the safe side. This result is illustrated with an example of comparisons for a four-car articulated vehicle in Fig. 4. The result is in contradiction to the

statement usually presented and shows that, when commenting the relation of linearized and non-linear stability assessment, we must always consider which parameters and criteria of both methods are to be applied.

The comparisons demonstrated that calculations linearized under conditions applied during railway vehicle engineering deliver rather conservative results. The linearized calculations are well suited for preliminary calculations, design particularly when no information complete concerning the actual contact geometry wheelset/track is available in an early project phase. The verification calculations should then be carried out with the aid of more non-linear exact analyses.



Fig. 4. Comparison of non-linear and linearized calculations: The critical speeds from linearized calculations for minimum damping 5% and Kalker's factor 1.0 (bottom diagrams) are lower than the results from non-linear analyses

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