

# WHEEL/RAIL CONTACT GEOMETRY PARAMETERS IN REGARD TO VEHICLE BEHAVIOUR AND THEIR ALTERATION WITH WEAR

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## ABSTRACT

This paper presents the progress of an attempt to identify wheel/rail contact geometry parameters related to vehicle behaviour. A so called “nonlinearity parameter” has recently been proposed with the aim to extend the commonly used characterisation of wheel/rail contact geometry by one equivalent conicity value. New investigations on a large amount of measured wheel and rail profiles show the typical alteration of the contact geometry parameters due to wear. The equivalent conicity increases while the nonlinearity parameter decreases with increasing vehicle mileage. This phenomenon can be explained by an increase of the contact conformity with wear. A new parameter for assessing the concentration of contact points during lateral wheelset displacement and thus indicating the wear spreading is proposed. The new parameters can support the assessment of contact geometry wheelset/track, the selection of wheel and rail profiles in simulations as well as the development of new wheel profiles.

## 1 INTRODUCTION

This paper presents new findings regarding the wheel/rail contact geometry parameters related to dynamic behaviour of railway vehicles as well as regarding their development due to wear with increasing running distance. Ref. [1] by one of the authors proposed to extend the commonly used contact geometry characterisation based on the equivalent conicity calculated for a wheelset displacement amplitude of 3 mm by a second parameter called **nonlinearity parameter (NP)**. However, no indication about the values of this parameter in service and its alteration due to wear of wheels and rails were presented. This paper describes recent investigations on a large amount of measured wheel and rail profiles and shows the typical development of the contact geometry parameters, in particular NP, due to wear. A hypothesis explaining the observed relationship in regard to the spreading of the contact between wheel and rail is presented and a new parameter for assessing the concentration of contact points is proposed. Suggestions for the usage of the newly proposed contact geometry parameters are presented.

## 2 WHEEL/RAIL CONTACT GEOMETRY PARAMETERS IN REGARD TO RUNNING DYNAMICS

Equivalent conicity [2] is traditionally used to assess the wheel/rail contact geometry in the railway practice and is widely recognised as a useful parameter in regard to running stability. However, the equivalent conicity as a linearised parameter does not consider the nonlinearity of wheel/rail contact geometry. The shape of the rolling radius difference function in the tread area can be either progressive, resulting in an increasing conicity (Type A in Figure 1), or degressive, resulting in a decreasing conicity (Type B in Figure 1). The newly proposed nonlinearity parameter (NP) [1] describes this property of the contact geometry wheelset-track. It is defined as the slope of the conicity function between the conicity value  $\lambda_2$  for the wheelset displacement amplitude of 2 mm and conicity value  $\lambda_4$  for the wheelset displacement amplitude of 4 mm:

$$NP = \frac{\lambda_4 - \lambda_2}{2} \quad (1)$$

The investigations in [1] as well as in [3], [4] and [5] demonstrated that this parameter influences the vehicle’s behaviour at the stability limit. If the vehicle system does not contain significant nonlinearities in vehicle suspension like friction

components or play in a wheelset guidance, which is usually fulfilled for modern railway rolling stock, the vehicle behaviour is predominantly influenced by the nonlinearities of the wheel/rail contact. This is illustrated in Figure 1 using two wheel/rail contact geometries with the same equivalent conicity for a wheelset displacement amplitude of 3 mm but different values of the nonlinearity parameter: a positive  $NP$  at contact geometry A and a negative  $NP$  at contact geometry B. The effect of  $NP$  on the vehicle behaviour can be described as follows:

- A positive  $NP$  results in a sudden change between the stable run and the limit cycle oscillations. This expresses itself in so called subcritical bifurcation, see the right diagram in Figure 1. Once the limit oscillation starts, it achieves large amplitudes often resulting in an exceedance of the safety limits according to standards for vehicle acceptance. Moreover, as a consequence of the contact nonlinearity, there is a speed range at which the vehicle can run at the same conicity either stable or in a limit cycle (co-existence of 2 solutions).
- A negative  $NP$  results in a gradual change of vehicle behaviour at the stability limit. The limit cycle starts at lower speed than in the previous case. The oscillation starts with small amplitude which increases with increasing speed; the bifurcation behaviour is supercritical.

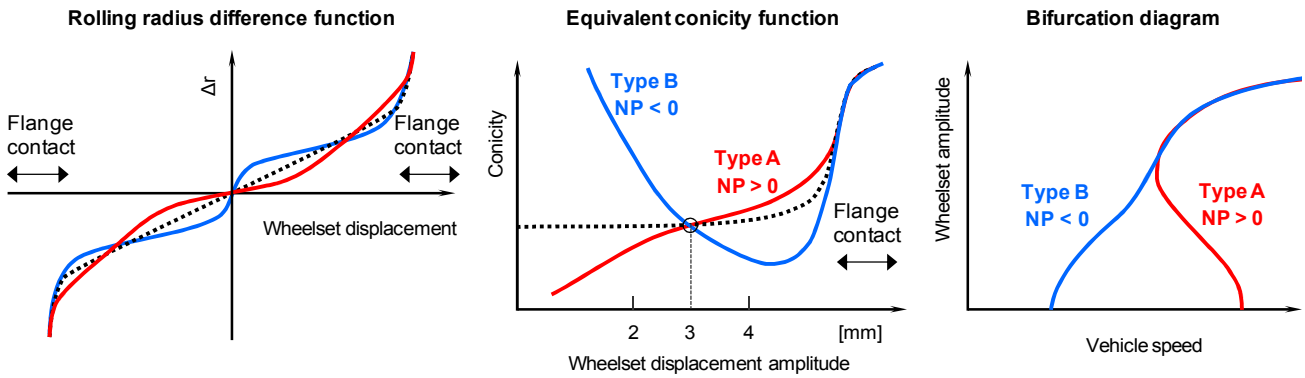


Figure 1: Rolling radius difference functions and the corresponding equivalent conicity functions for wheel/rail contact geometries with the same equivalent conicity for a wheelset displacement amplitude of 3 mm. Effect of  $NP$  on the bifurcation diagram displaying the amplitude of the limit cycle of wheelset oscillation.

The described phenomenon has not only impact on the stability assessment, but influences also the vehicle's behaviour at speeds below the critical speed in simulations using measured track irregularities as demonstrated in [1]. The same observation is mentioned also in [6].

### 3 ALTERATION OF WHEEL/RAIL CONTACT GEOMETRY PARAMETERS WITH WEAR

The evaluation of several thousands of measured wheel profiles [7] allowed an identification of the relationship between the typical alteration of wheel/rail contact geometry due to wheel wear with vehicle mileage and the proposed characteristic parameters. The evaluation of measured wheel profiles combined with nominal rail profiles shows that the equivalent conicity increases while  $NP$  decreases with increasing vehicle running distance as it is illustrated on the example of German high speed trains ICE 3 in Figure 2. The diagrams present the measurements from 9030 wheelsets conducted in 2003. The calculation of equivalent conicity is carried out combining the measured wheel profiles with nominal track conditions, i.e. track gauge 1435 mm and rail profile 60E1 [8] (formerly UIC 60) with the inclination 1:40 which were used at that time as nominal conditions on the most German lines. An increase of the equivalent conicity due to wear with increasing vehicle mileage is provided with a decrease of  $NP$ , so that the contact geometry of worn wheel profiles running on new rail profiles is characterised by a negative  $NP$  value of  $-0.04 \div -0.03$ .

A multi-national survey of wheel/rail contact geometry conducted in some European countries in the framework of the EU founded research project Dynotrain [7] demonstrated that the increase of equivalent conicity together with the decrease of  $NP$  can be observed also in the evaluations combining worn rail and corresponding worn wheel profiles (worn in networks with the same nominal rail inclination). The examples in Figure 3 displaying the profile analysis in two speed categories (120-160 km/h and 160-230 km/h) demonstrate that  $NP$  in average decreases with increasing equivalent conicity. Linear regression is applied to describe the relationship between the equivalent conicity and  $NP$  due to wheel wear with increasing mileage

$$NP = NP_0 - k\lambda \quad (2)$$

with  $\lambda$  – equivalent conicity

$k$  – regression coefficient

$NP_0$  – initial NP value (new profile).

The linear regression provides negative regression coefficient  $k$  with values between -0.11 and -0.04. Although the spread of values presented in the diagrams is huge so that the regression lines might have little meaning, the analyses demonstrate the same tendency occurring in all countries and speed categories. The evaluation shows that the basic relationship between wear, equivalent conicity and  $NP$  as observed on wheels is valid also for the combination of wear on both wheels and rails.

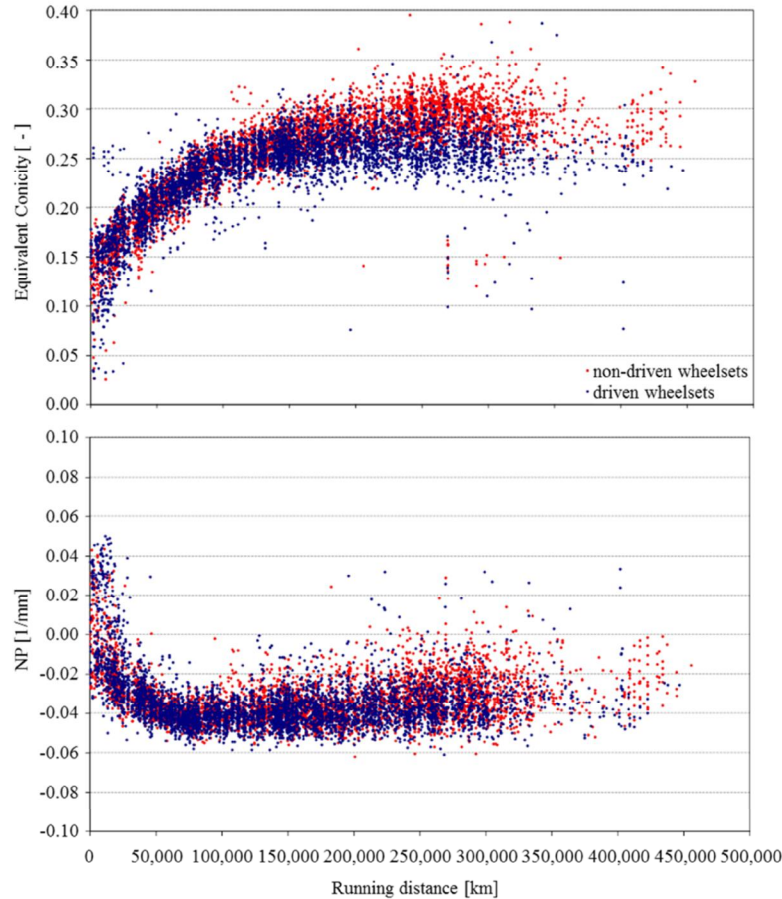


Figure 2: Development of equivalent conicity and  $NP$  with wear (i.e. vehicle mileage) – example of fleet ICE 3, 9030 wheelset measurements; evaluation of measured wheel profiles in combination with rails 60E1 1:40.

## 4 RELATIONSHIP BETWEEN WHEEL/RAIL CONTACT GEOMETRY PARAMETERS AND CONTACT SPREADING

### 4.1 Analysis of measured wheel profiles of a German high speed train

The described phenomenon can be explained by an increase of contact conformity with running distance. With increasing conformity, the contact between wheel and rail is spread more widely across the wheel and rail profiles. This leads to large changes of the rolling radii at very small wheelset displacements around the nominal position resulting in increasing equivalent conicity and decreasing  $NP$ . Another consequence of the conformal contact is that the wheel profile remains form stable because wear is more widely spread over the tread area.

To confirm this hypothesis, a set of measured wheel profiles of a German ICE 2 high speed train was evaluated in detail. The wheel profile data used for this evaluation were sampled systematically during a test of different wheel profile designs in 1998. The contact geometry analyses performed by means of the computer code RsGeo [9] provide the horizontal coordinate  $y_C$  of the contact point on the wheel profile in function of the lateral wheelset displacement  $y_{WS}$ . The presented investigations were carried out using elastic contact of wheel and rail, resulting in a contact patch. The term “contact point” is used here for the point of the geometric centre of the wheel/rail contact patch. In case of more than one contact patch, the main contact with the largest contact patch area is used for the evaluation.

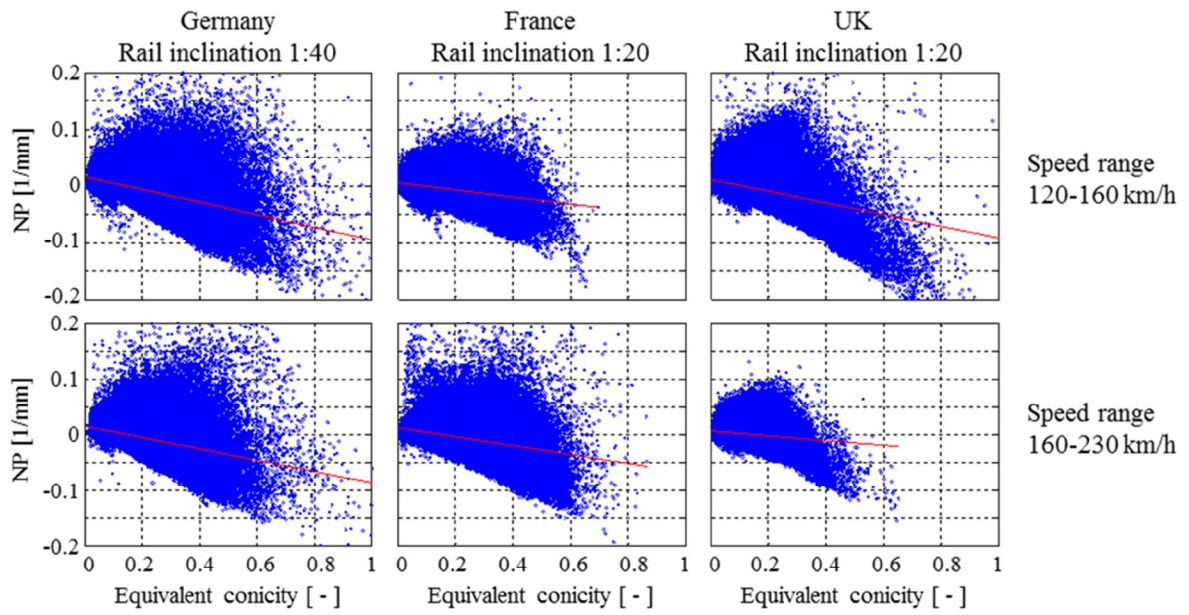


Figure 3: Relationship between  $NP$  and equivalent concity evaluated using worn wheel and worn rail profiles in different European countries and speed categories.

Figures 4 - 6 present examples of evaluation results for one wheelset of the dining car which was equipped with the reference wheel profile S1002 with reduced flange thickness resulting to a nominal distance of active faces of 1423 mm. The wheel profiles were measured starting from newly turned profiles up to the running distance of 239,000 km. Figure 4 shows the alteration of the difference of rolling radii and the equivalent concity functions with vehicle's mileage for the selected ICE 2 wheelset evaluated in combination with nominal rail profile 60E1 with inclination 1:40. One can see that the slope of the difference of rolling radii around the centred position increased with running distance and the equivalent concity function changed its shape from Type A (with a positive  $NP$ ) to Type B (negative  $NP$ ).

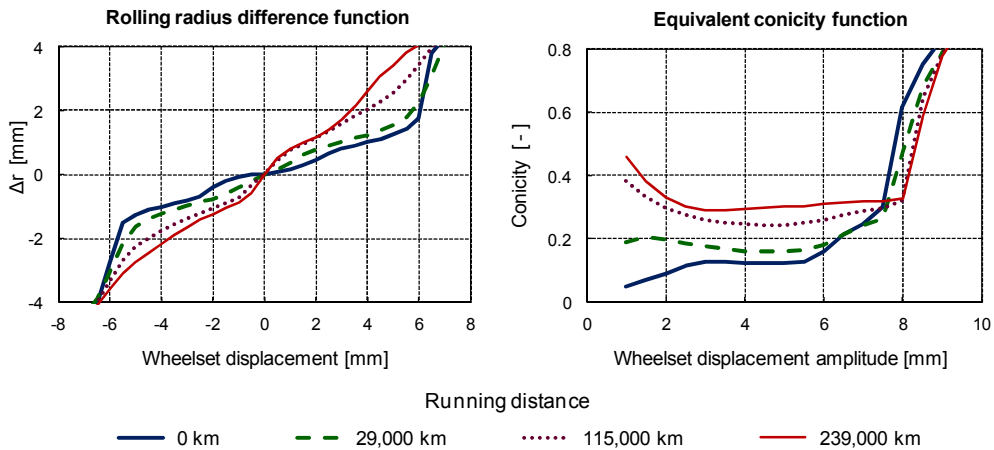


Figure 4: Measured ICE 2 wheel profiles evaluated in combination with nominal rail profile 60E1 1:40 and track gauge 1435 mm. Alteration of the functions difference of rolling radii and equivalent concity with vehicle's mileage.

Figure 5 shows an analysis of the contact point location. The left diagram in Figure 5 displays the contact point movement  $d_{y_C}$  over the lateral wheelset displacement  $y_{WS}$ . The contact point movement represents the derivation of the function of contact position on the wheel profile  $y_C = f(y_{WS})$ . It is calculated here numerically as the shift of the contact point position  $\Delta y_C(y_{WS})$  related to the change of lateral wheelset displacement  $\Delta y_{WS}$ .

$$dy_c(y_{ws}) = \left| \frac{\Delta y_c(y_{ws})}{\Delta y_{ws}} \right| = \left| \frac{y_c(y_{ws} + \Delta y_{ws}) - y_c(y_{ws})}{\Delta y_{ws}} \right| \quad (3)$$

The middle diagram in Figure 5 presents the so called contact bandwidth change rate as proposed in [10]. This parameter is defined assuming a lateral wheelset movement with the amplitude  $A_{ws}$ . The distance between the contact point location for the wheelset displacement  $y_{ws} = -A_{ws}$  (to the left) and the contact point position for  $y_{ws} = A_{ws}$  (to the right) is defined as contact bandwidth  $L_w$

$$L_w(A_{ws}) = y_c(-A_{ws}) - y_c(A_{ws}) \quad (4)$$

The ratio of the contact bandwidth and the respective lateral wheelset displacement is defined as contact bandwidth change rate  $dL_w$

$$dL_w(A_{ws}) = \frac{L_w}{2A_{ws}} \quad (5)$$

The right diagram in Figure 5 shows a new parameter called contact concentration. This parameter characterises the frequency of the contact point occurrence across the wheel profile and provides an indication of wear distribution in dependency on the lateral wheelset displacement. The determination of this parameter is based on the following assumptions:

- straight track with stochastic track irregularities,
- stochastic lateral wheelset displacement with normal distribution with standard deviation  $\sigma$  (here  $\sigma = 2.5$  mm selected).

It is assumed that the local wear of wheels and rails is related to the local frequency of contact point occurrence because wear is higher in the area with a more frequent contact occurrence as presented in [11]. Moreover, a wide spread (i.e. low concentration) of the contact points between wheel and rail is usually correlated with conformal contact and thus larger contact patch size, lower normal stress and consequently lower wear and vice versa, supporting the assumption that wear is proportional to the concentration of the contact point occurrence.

The parameter contact concentration is defined as a reciprocal value of the contact point movement  $dy_c$ , multiplied with the respective percentile  $p_{y_{ws}}(y_{ws})$  of the wheelset displacement occurrence

$$c_c(y_{ws}) = \frac{p_{y_{ws}}(y_{ws})}{dy_c(y_{ws})} \quad (6)$$

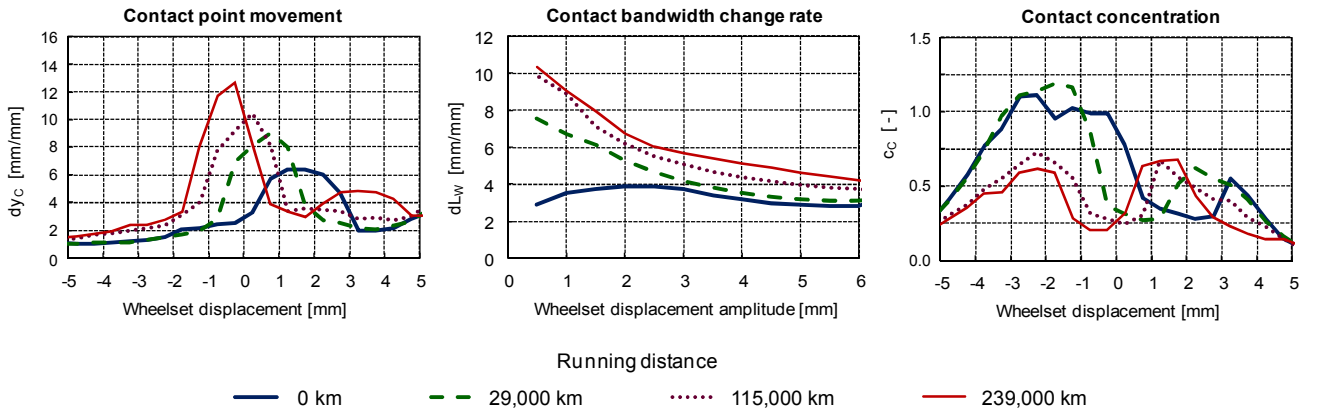


Figure 5: Diagrams of contact point movement (according to Equation 3), contact bandwidth change rate (according to Equation 5) and contact concentration (according to Equation 6) of measured ICE 2 wheel profiles (left wheel), evaluated in combination with nominal rail profile 60E1 1:40.

The evaluation results illustrated exemplarily in Figure 5 confirm the hypothesis about the relationship between contact geometry parameters, contact spreading and wear. It can be seen, that the contact point movement increases with wear (left diagram in Figure 5). The largest contact point movement occurring at new wheel profiles for wheelset displacements between 1 and 2 mm is approximately doubled at worn wheel profiles and shifted closer to the centred wheelset position, with a peak at wheelset displacements between -1 and 0 mm. A contact point movement increasing with wear results in a wider spread of wear in rolling contact. This is indicated by the increase of contact bandwidth change rate with wear, particularly for small wheelset displacement amplitudes just after re-profiling, see middle diagram in Figure 5. The wider spreading of contact points at worn wheels is documented in the right diagram of Figure 5 by decreasing contact

concentration, particularly for small wheelset displacements close to zero.

To characterise the average contact concentration properties of the respective combination of wheel and rail profiles by one value, a new parameter **contact concentration index (CCI)** is introduced, calculated by averaging the contact concentration  $c_C(y_{WS})$  over the normal distribution between  $-3\sigma$  and  $3\sigma$  of lateral wheelset displacement with the selected standard deviation  $\sigma$

$$CCI = \frac{1}{n} \sum_{i=1}^n \frac{p_{y_{WS}}(y_{WS_i})}{dy_C(y_{WS_i})} \quad (7)$$

with  $n$  – number of distribution classes.

The contact concentration index  $CCI$  indicates the concentration of contacts and thus the concentration of wear for the respective combination of wheel and rail profiles by one value, however, without any quantitative information about the level of wear.

Figure 6 presents the development of equivalent conicity,  $NP$  and  $CCI$  for the measured ICE 2 wheel profiles of the wheelset used in the evaluation discussed above. The diagrams demonstrate similar evolution of the contact geometry parameters as in the previous results displayed in Figures 2 and 3. The equivalent conicity increases while  $NP$  decreases. The equivalent conicity is indirectly proportional to  $CCI$ . This confirms the presented hypothesis that the conicity increase is accompanied with wider contact spreading, i.e. lower contact concentration. It moreover demonstrates that  $CCI$  is a useful parameter representing the level of contact conformity.

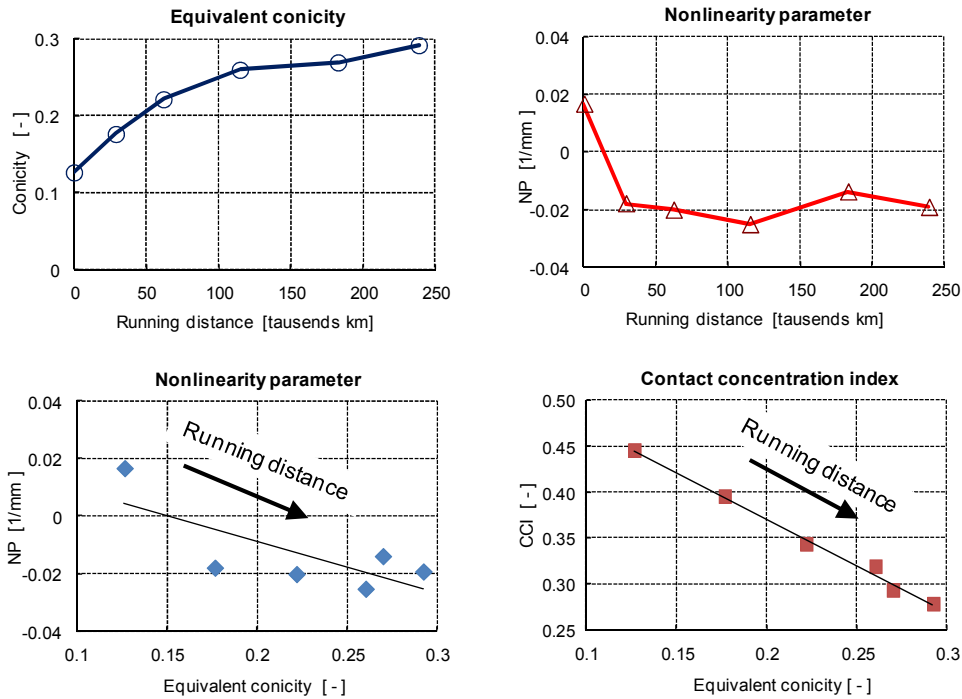


Figure 6: Relationships between equivalent conicity, nonlinearity parameter ( $NP$ ) and contact concentration index ( $CCI$ ) of measured ICE 2 wheel profiles with the vehicle's running distance (evaluated in combination with nominal rail profile 60E1 1:40 and track gauge 1435 mm).

## 4.2 Comparison with other publications

Recent works published by other authors confirm the presented findings. Ref. [12] presents the development of equivalent conicity with mileage of high speed vehicles in China. Figure 7 shows the alteration of equivalent conicity with mileage of the wheels turned to design profile LMA. The equivalent conicity increases rapidly during the first period of 50,000 km running distance. The originally small lateral contact bandwidth increases during this initial period as can be seen in Figure 7c. The increase of the contact spreading is accompanied with a gradual increase of the equivalent conicity for a wheelset displacement amplitude of 3 mm. The conicity function obtains a declining form with  $NP$  of about -0.08 at the equivalent conicity of 0.32, i.e. the same tendency as demonstrated on ICE high speed trains in Germany.

The presented phenomenon is independent of the shape of the initial wheel profile. This can be seen comparing the previous results with the results shown in [13] displaying the wheel wear of CRH3 trains in China using the wheel profile S1002CN

as initial profile. Here, the hollow wear starts at 42,000 km. At this mileage, the equivalent conicity increased from 0.17 to 0.31, and  $NP$  achieved a value of about -0.06. Another confirmation of the observed phenomena can be found in Ref. [14] presenting the typical worn wheel profiles measured on Italian high speed vehicles (nominal wheel profile S1002 on rail 60E1 with inclination 1:20). The equivalent conicity of those measured worn profiles lays between  $0.3 \div 0.4$  and  $NP$  reaches values of about -0.07, again confirming the previously described observations.

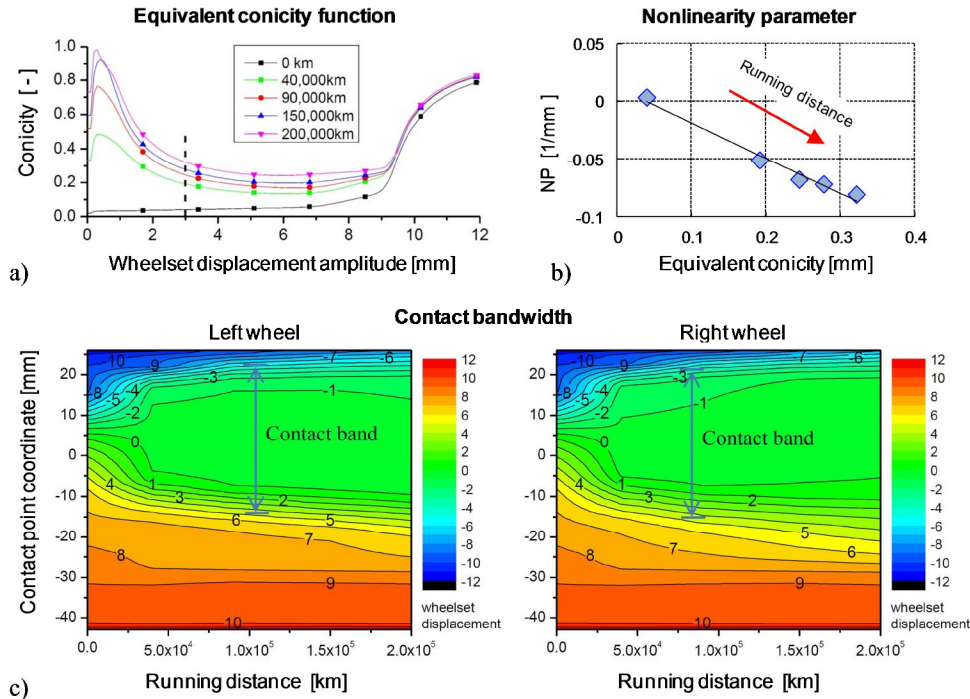


Figure 7: Alteration of contact geometry wheelset/track with mileage on high speed vehicles with LMA wheel profile in China (from [12]):

- Change of equivalent conicity function with mileage,
- $NP$  over equivalent conicity,
- Change of contact band between wheel and rail with mileage.

### 4.3 Comparison of design wheel profiles

The presented contact geometry parameters can be used to assess proposals for new design wheel profiles. The design of new profile shapes can be based on various approaches and targets [15], [16], [17], [18], [19]. However, the shape of the wheel profile will change with wear. Our investigations confirm that the alteration of the wheel profile shape in service will be reduced using a wheel/rail contact geometry with high conformity and thus large contact spreading. A comparison of so called “wear adapted profiles” with other design profiles supports this statement.

The following “wear adapted profiles” are evaluated:

- Wheel profile EN 13715 - S1002 / h28 / e32.5 / 6.7% [20], well known as S1002, was proposed by the expert committee ORE S1002 in 1973. Although the committee considered experience from different European countries, the tread shape of the S1002 wheel profile is very close to the wear adapted profile developed in Germany based on experience with vehicles in service on tracks with rail inclination 1:40.
- Wheel profile P8 BR – introduced in UK, developed based on experience with vehicles in UK in service on tracks with nominal rail BS 113A and rail inclination 1:20.

Other compared wheel profiles are:

- Wheel profile EN 13715 - 1/40 / h28 / e32.5 / 15% [20] – conical wheel profile with tread inclination 1:40, in combination with rail 60E1 installed with inclination 1:20 (Remark: Rail inclination has no influence on the contact geometry in case of conical wheel profiles).
- Wheel profile EN 13715 - S1002 / h28 / e32.5 / 6.7% [20] in combination with rail 60E1 installed with inclination 1:20 – this combination is used in some countries, but is known as not form stable; the wheel profile shape changes after rather small mileage.

- Wheel profile EN 13715 EPS - / h28 / e32.5 / 10% [20] – wheel profile with the tread shape identical to P8 BR but with flange thickness of 32.5 mm instead of 30 mm; here in combination with rail 60E1 with inclination 1:40.

Figure 8 shows the contact point movement and the contact bandwidth change rate of the compared design wheel profiles. Only the wear adapted profiles show large contact point movement and a high contact bandwidth change rate. It is interesting to see that the contact bandwidth change rate curves for both examples of wear adapted profiles are nearly identical in spite of different shapes of those wheel and rail profiles as well as different rail inclinations. The new parameter *CCI* of the wear adapted profiles is much lower than that of the conical wheel profile and of the wheel profile S1002 combined with the rail 60E1 inclined by 1:20.

The wheel profile EPS used in this comparison possesses the same shape of the tread area as the profile P8 BR but a different flange thickness. In combination with the rail profile 60E1 and rail inclination 1:40 the performance of this wheel profile is in between the “wear adapted profiles” and the conical profile. The contact bandwidth change rate is in this case slightly higher than that of the conical profile.

This experience can be used to assess newly developed design wheel profiles. The profile will be form stable, i.e. the tread part of the wheel profile will keep its shape, if the wheel/rail profile combination provides a large contact bandwidth change rate (approximately above 4 for wheelset amplitudes lower than 3 mm) and a small *CCI*. Otherwise the profile shape will change after a rather short running distance.

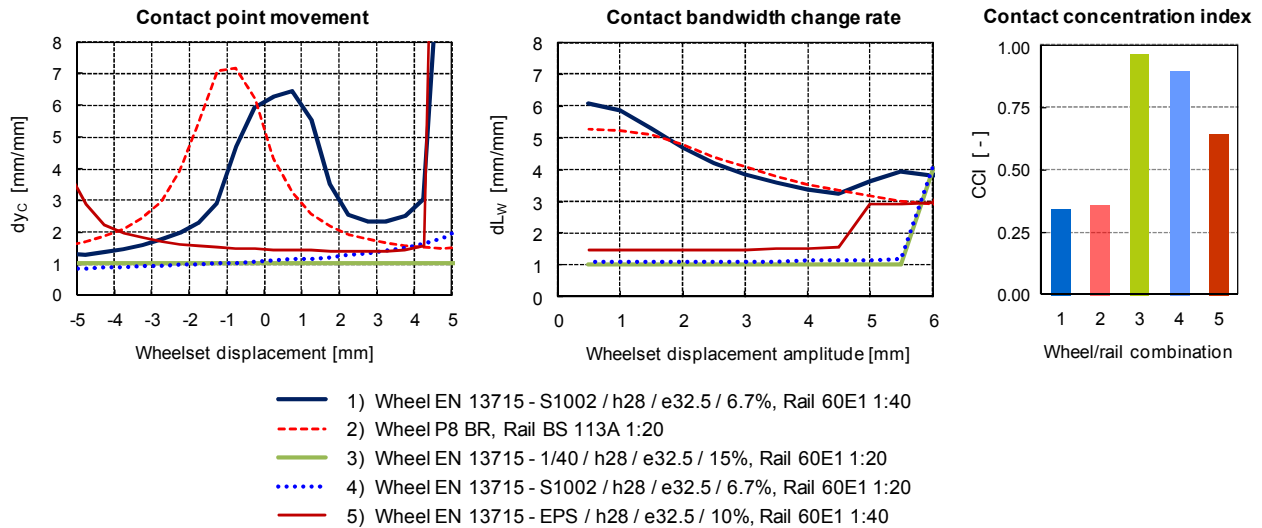


Figure 8: Contact point movement (left), contact bandwidth change rate (middle) and contact concentration index (right) of the selected wheel/rail profile combinations (track gauge always set to 1435 mm).

## 5 CONCLUSIONS

The equivalent conicity is commonly used to assess the wheel/rail contact geometry. As it is a linearised parameter, it is a severe simplification. One conicity value can represent an infinite set of wheel and rail profiles. This paper presents new parameters proposed to support the contact geometry assessment. It provides insight into the relationship between the equivalent conicity, the proposed contact geometry parameters and the profile alteration due to wear with increasing mileage.

The nonlinearity parameter (*NP*) proposed in [1] extends the characterisation of wheel/rail contact geometry using the equivalent conicity for a wheelset displacement amplitude of 3 mm by an auxiliary information related to vehicle dynamic behaviour as well as profile wear condition. This parameter can be applied for

- assessment of profile measurements with the aim to improve the characterisation of wheel/rail contact geometry in regard to the vehicle dynamic behaviour,
- use of profile combinations in simulations to allow better selection of wheel and rail profiles intended to be used as representative contact geometries and
- design of new wheel profiles because it provides an indication of profile shape stability: profile combinations with negative *NP* can be expected to keep their form longer than those with positive *NP*.

Starting with a new wheel profile, the equivalent conicity usually at first increases while *NP* decreases with increasing mileage. This increase of equivalent conicity is provided by increasing conformity and larger contact spreading. Consequently, the wheel profile shape gets more form stable at high mileage, and the conicity and *NP* remain nearly constant.



A new parameter called contact concentration index (*CCI*) is proposed to assess the contact conformity. It is applied here in the evaluations of measured wheel profiles as well as of design wheel profiles together with the contact bandwidth change rate proposed in [10]. Both parameters are related to the development of the equivalent conicity due to wear. The profile combinations with high contact bandwidth change rates and small *CCI* will be more form stable, i.e. the tread part of the wheel profile will keep its shape. These parameters can thus be used to assess new proposals for wheel and rail profiles regarding their wear in service.

A large flange wear on curvy lines may certainly change the profile shapes and also the characteristic wheel/rail contact geometry parameters in a different manner than explained in this paper. Such cases, however, represent nowadays a rather small percentage of railway service because of increasing number of high speed lines with predominantly straight tracks. This stresses the importance of the presented results for railway practice.

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