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Vehicle System Dynamics

Characteristic parameters of non-linear wheel/rail contact geometry

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The equivalent conicity is widely used to characterise the wheel/rail contact geometry; however, it does not consider the contact non-linearity. There is a need for an improved but still simple description which accounts for the most important effect of the contact non-linearity on the running dynamics of railway vehicles. This article demonstrates the influence of the contact non-linearities on the behaviour of railway vehicles at the stability limit and presents a description of wheel/rail contact geometry using two parameters. The proposed characteristic parameters are compared on examples of wheelset/track pairs and the correlation between the proposed parameters and vehicle behaviour presented.

Keywords***:*** wheel/rail contact geometry, equivalent conicity, stability, critical speed

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1. Introduction

The contact geometry between wheel and rail or wheelset and track, respectively, has an important influence on the running dynamics of railway vehicles. The actual contact geometry changes not just due to variation of rail profiles on different track sections but also due to variation of track gauge, rail inclination, rail irregularities in vertical and lateral directions, flexibility of rail support, as well as variation of wheelset back-to-back distance due to axle bending.

The main parameters influencing the contact geometry are the wheel and rail profiles, rail inclination, wheel back-to-back distance and track gauge. These parameters are typically used for an assessment of the contact geometry or as the input data for multi-body simulations of railway vehicle dynamics. Although progress in measuring systems allows a continuous rail profile measurement along the track, an application of the continuously measured rail profiles is not the state-of-the-art in non-linear multi-body simulations yet.

Because of the wide scatter of the wheel/rail contact geometry, simplified parameters are required to assess the contact geometry. The equivalent conicity – a parameter originating from the linearisation of the system wheelset/track – is widely used in the railway community to characterise the contact geometry conditions. This parameter is introduced in EN 14363 [1] and UIC Code 518 [2] for the assessment of wheel/rail contact geometry during the testing for vehicle acceptance. It is also used in Technical Specifications for Interoperability [3–5] to characterise the track (combining the measured rail profiles with theoretical wheel profiles) or the geometry of worn wheel profiles (combining the measured wheel profiles with theoretical rail profiles), respectively.

During the last decades, the progress in railway vehicle non-linear dynamics has contributed to better understanding of the influence of the wheelset/track non-linearities on vehicle behaviour; a number of papers present investigations related to the influence of non-linear wheel/rail contact geometry on the stability of railway vehicles. The parameters used to change the wheel/rail contact geometry are usually the wheel and rail profiles, inclination of rails and track gauge [6–9]. These parameters, however, do not allow any generalised conclusion regarding the relationship between the wheel/rail contact non-linearity and the vehicle's behaviour. A simplified characterisation of the wheelset/track contact geometry remains an important topic for the assessment of tracks, vehicles and input for multi-body simulations. There is a gap between the progress of non-linear dynamics and the practical assessment of wheel/rail contact geometry. This article attempts to start the work to overcome this gap. It shows the effects of wheel/rail contact non-linearities and proposes a new characterisation of wheel/rail contact geometry consisting of two parameters.

The paper is organised as follows: the traditional characterisation of wheel/rail contact geometry using the quasi-linearisation is described in chapter 2; chapter 3 demonstrates the effect of non-linearity of the wheelset/track contact geometry on vehicle behaviour at the stability limit; chapter 4 presents the proposed definition of new parameters characterising the contact non-linearity. This description is compared on six examples of wheelset/track contact geometries with three different levels of equivalent conicity. The relationship between the characteristic parameters, the bifurcation at the stability limit and the dynamic behaviour of a vehicle running on track with measured irregularities is shown. A summary and an outlook regarding further investigations are given in chapter 5.

2. Characterisation of wheel/rail contact by linearisation

2.1. Quasi-linear wheel/rail contact model

The equivalent conicity represents an established assessment criterion for the wheelset/track contact geometry. The most widely used quasi-linear wheel/rail contact model [10] consists of three parameters

- equivalent conicity λ
- contact angle parameter ε
- roll parameter σ.

The linearisation of profiles approximated by circular arcs describes these linearisation parameters in terms of wheel profile radius R_W, rail profile radius R_R, contact angle δ_0 in nominal position, a half of the tape line distance e_0 and the nominal wheel radius r_0, see Figure 1.

Applying a linearisation around the nominal position, the equivalent conicity λ reads [11]

$$\lambda = \frac{R_W \cdot \sin\delta_0}{R_W - R_R} \cdot \frac{e_0 + R_R \sin\delta_0}{e_0 \cos\delta_0 - r_0 \sin\delta_0} \tag{1}$$

The difference of the contact angles on the left and right wheel is described by the contact angle parameter ε

$$\varepsilon = \frac{e_0}{R_W - R_R} \cdot \frac{e_0 + R_W \sin\delta_0}{e_0 \cos\delta_0 - r_0 \sin\delta_0} \tag{2}$$

The roll angle around the longitudinal axis is characterised by the roll parameter σ which reads

$$\sigma = \frac{e_0 \sin\delta_0}{e_0 \cos\delta_0 - r_0 \sin\delta_0} \tag{3}$$

Assuming a small contact angle ($\sin\delta_0 \approx \delta_0$, $\cos\delta_0 \approx 1$) and considering the term $\frac{e_0 + R_R\delta_0}{e_0 - r_0\delta_0}$ which is very close to 1 as equal to 1, the Equation (1) for the conicity linearised around the nominal contact point can be simplified into

$$\lambda = \delta_0 \left(1 + \frac{R_R}{R_W - R_R} \right) \tag{4}$$

where the term $\frac{R_R}{R_W - R_R}$ can be called conformity parameter as it is related to the conformity of wheel and rail arc sections.

From the Equation (4) one can observe that the value of equivalent conicity is influenced by:

- contact angle
- conformity of wheel and rail profile.

The equivalent conicity can be raised either by high contact angle or high conformity. A high contact angle together with low conformity is connected with a rather small lateral shift of the contact area across the wheel and rail profiles (see Figure 2), rather small contact angle difference and a large roll angle. A high conformity together with low contact angle result in a large contact shift along with large contact angle difference and small roll angle. A more conformal contact can be often observed on worn wheel and rail profiles. However, there is no correlation between the contact geometry and either new (theoretical) or worn profiles.

The parameters of the quasi-linear wheelset/track contact model are calculated by harmonic linearisation [11], whereby this linearisation considers not only a very small wheelset displacement around the nominal position, but the specified wheelset displacement inside the clearance between wheelset and track. A sensitivity of the critical speed calculated using quasi-linear wheel/rail contact to the variation of contact angle parameter and roll parameter was presented by the author in [12]. All in all, the sensitivity of the running stability to the contact angle parameter and roll parameter is low compared with the sensitivity to the equivalent conicity. The equivalent conicity influences the other parameters, so that the conicity is the only parameter usually mentioned in frame of the wheel/rail contact geometry assessment.

The value of equivalent conicity for a wheelset amplitude of 3 mm is typically used to describe characteristic properties of wheel/rail contact geometry in railway applications [1,2]. Therefore, if there is no other reference, the equivalent conicity is understood as the conicity for 3 mm amplitude.

There are several definitions and methods for conicity calculation. The wheelset movement considered in the calculation of the equivalent conicity is either periodic or stochastic [13]. While a stochastic movement with the specified standard deviation is used to calculate equivalent conicity in UK, a periodic wheelset movement is traditionally used in continental Europe, and has been introduced in the standards dealing with the determination of equivalent conicity [14,15].

The methods frequently used to calculate the equivalent conicity are

- harmonic linearisation
- equivalent linearisation by the application of Klingel formula
- linear regression of the rolling radii difference function.

When the wheelset with conical tread profiles moves laterally with a displacement y from its centred position, the rolling radii of the right wheel r_r and left wheel r_l are different. The conicity λ of the wheel tread can be then expressed as a function of wheelset rolling radii difference Δr

$$\lambda = \frac{r_r - r_l}{2y} = \frac{\Delta r(y)}{2y} \tag{5}$$

The rolling radii difference Δr in function of lateral wheelset displacement y is used for the evaluation of equivalent conicity. The principles of the aforementioned equivalent conicity calculation methods are presented in the following subchapters. The presented analyses assume that left and right profiles of both wheels and rails are symmetric and wheel radii of both wheels are identical. These assumptions lead to symmetric contact geometry functions.

2.2. Harmonic linearisation

This method [11] aims for the minimum quadratic error between the expected value of the non-linear function $\Delta r(y)$ and the expected value of the quasi-linear approach $\Delta r = k\,y$ when integrating over one period of wheelset motion. With an approach of harmonic wheelset movement with an amplitude A

$$y(\omega t) = A\sin(\omega t) = A\sin\varphi \tag{6}$$

the equivalent conicity yields after the integration as a non-linear function of the linearisation amplitude A [11]

$$\lambda(A) = \frac{1}{2\pi A}\int_0^{2\pi} \Delta r(A\sin\varphi)\sin\varphi \, d\varphi \tag{7}$$

For measured profiles of wheels and rails presented as a set of numbers, the equivalent conicity is calculated by numerical integration of Equation (7).

2.3. Equivalent linearisation

The equivalent linearisation [14,15] is based on determining the wavelength L of wheelset motion and calculating the equivalent conicity by the Klingel formula

$$\lambda = \left(\frac{2\pi}{L}\right)^2 e_0 r_0 \tag{8}$$

The equation of motion of a free, massles wheelset which rolls on a straight, ideal, rigid track in direction x with speed v is derived assuming that the spin and the lateral component of normal forces in the wheel/rail contact are neglected. Considering yaw movement ψ around the vertical axis with small angle, the equation of free motion of this wheelset in the horizontal plane results from the conditions of zero longitudinal and lateral wheel/rail creepages

$$\frac{e_0}{v}\frac{d\psi}{dt} + \frac{\Delta r(y)}{2\, r_0} = 0 \tag{9}$$

$$\frac{1}{v}\frac{dy}{dt} - \psi = 0 \tag{10}$$

Eliminating the time these equations can be written as

$$\psi\frac{d\psi}{dy} + \frac{\Delta r(y)}{2e_0 r_0} = 0 \tag{11}$$

$$dx = \frac{dy}{\psi} \tag{12}$$

The equivalent linearisation method described in [14,15] applies a numerical integration of Equation (11) to obtain the movement of the wheelset's centre of gravity starting from y_{min} (for which $\psi_{min} = 0$) and going up to y_{max} (for which $\psi_{max} = 0$). This integral needs only to be calculated once for sufficiently large amplitudes of wheelset displacement to cover the investigated range. The integration of Equation (12) is then used to calculate the wave length L for each of the wheelset amplitudes.

The wave length L can also be calculated directly from the following equation obtained inserting (12) into (11)

$$\frac{d^2 y}{dx^2} + \frac{\Delta r(y)}{2 e_0 r_0} = 0 \tag{13}$$

by integration for initial conditions $y(0) = y_{min}$, $y'(0) = 0$ (i.e. $\psi = 0$).
The equivalent conicity is calculated inserting the wave length L into Equation (8).

2.4. Linear regression of the rolling radii difference function

This linearisation method [14,15] is based on the fact that for a linear Δr function the slope k of this function is equal to 2λ

$$\Delta r = 2\lambda\, y = k\, y \tag{14}$$

For a non-linear function $\Delta r = f(y)$ the slope of a linear regression gives an approximation of 2λ. Hence, the equivalent conicity is obtained calculating the linear regression of the Δr function in the range between y_{min} and y_{max} and reads

$$\lambda = \frac{k_{LR}}{2} \tag{15}$$

where k_{LR} is the slope of the regression of the Δr function.

2.5. Comparison of linearisation methods

Due to the different conicity calculation methods, the equivalent and harmonic linearisations introduce higher frequency of occurence of large lateral wheelset displacements than small ones. The rather equal occurrence distribution of the lateral wheelset displacements takes place for the linear regression method, whereas for the UK-method assuming a stochastic wheelset displacement, there is the highest frequency occurrence of small wheelset displacements. Different methods can certainly lead to different calculated conicity values. This topic has not been analysed in publications yet; the only publication known to the author about the differences between the different conicity calculation methods is the article by Bonadero [16].
The application of the equivalent conicity for a wheelset amplitude of 3 mm to characterise the wheelset/track contact geometry assumes linear contact geometry relations. The rolling radii difference and also the equivalent conicity are, however, influenced by the non-linearity of the wheel/rail contact geometry. Consequently, the vehicle behaviour can differ even when wheel/rail contact geometries posses the same equivalent conicity for the specified amplitude. The following investigations concentrate on the influence of the contact geometry non-linearities on the equivalent conicity. The aim of this contribution is to propose parameters suited to characterise the contact geometry from the point of view of the effect of its non-linearity on railway vehicle dynamics.

3. Effect of wheel/rail contact non-linearity on railway vehicle dynamics

It is well known that the wheel/rail contact geometry has an important influence on the running stability of railway vehicles. A detailed stability assessment of a non-linear system can be achieved by bifurcation analysis. In case of railway vehicles, the bifurcation diagram displaying the amplitude of wheelset lateral displacement is typically used to assess the vehicle's stability, see e.g. [17]. The author's investigations as well as comparison with other publications have allowed an identification of the interrelationship between the shape of the bifurcation diagram and the wheel/rail contact non-linearity represented by the equivalent conicity as a function of wheelset displacement amplitude. This relationship can be seen in Figure 3 for three different vehicles and two different wheel/rail contact geometries. Both examples of wheel/rail contact geometry represent the same equivalent conicity for 3 mm amplitude, but different values for other wheelset amplitudes. For the contact geometry A, there is progressive equivalent conicity in function of wheelset amplitude. The bifurcation analysis displays a subcritical Hopf bifurcation. In contrast, for the contact geometry B there is strongly declining equivalent conicity function for amplitudes up to 5 mm (i.e. in the wheel tread away from flange contact). A supercritical Hopf bifurcation can be observed for this wheel/rail contact geometry. Such a different dynamic behaviour on contact geometries with the equivalent conicity function of "Type A" and "Type B" was described for the first time in [18] and outlined more in detail in [17] and [19].

A wheel/rail contact with rather low conicity at small wheelset amplitudes and a positive slope of equivalent conicity function typically results in a sudden occurrence of a limit cycle with large amplitude leading to an exceedance of the safety limit. This behaviour is represented by subcritical Hopf bifurcation: the non-linear critical speed and the exceedance of the safety limit are very close due to a sudden appearance of the oscillation. A wheel/rail contact resulting in high conicity for small wheelset amplitudes and a negative slope of the equivalent conicity function usually leads to the limit cycle with an amplitude slowly growing with increasing speed. The bifurcation analysis delivers supercritical Hopf bifurcation. In this case, a limit cycle with small amplitude usually occurs at speeds far below the safety limit according to EN 14363 [1] and will not necessarily lead to an exceedance of the instability limit, see Figure 4.

The non-linearity of the contact geometry often determines the type of the Hopf bifurcation of railway vehicles as shown in the presented examples. The non-linearity of a vehicle model can supersede the abovementioned effect of wheel/rail contact and eventually also change the type of Hopf bifurcation as described in [20]. The tendency of the modification of bifurcation diagram will, however, remain similar.

The bifurcation diagram showing the subcritical Hopf bifurcation can indicate the risk to overestimate the critical speed. The vehicle will run stable even at speeds higher than the non-linear critical speed, if the track irregularities during the tests or the irregularity data applied in simulations are too low. A system showing the supercritical Hopf bifurcation possesses non-linear critical speed which is lower than the speed at which the safety limits are reached. An assessment of such a system can deliver too low critical speed with the criteria below the safety limits specified for vehicle acceptance, so the stability assessment can either underestimate or overestimate the safety relevant critical speed. Hence, an understanding of vehicle behaviour at the stability limit is an important part of railway vehicle stability assessment.

Whereas one can observe a correlation between the shape of the wheel/rail contact geometry functions and the shape of the bifurcation diagrams, no clear correlation could be confirmed between the wear of wheels and rails and the shape of the bifurcation diagrams either by the author or by Chung and Shim [21]. Hence, it is important to search for characteristic parameters of wheel/rail contact geometry which express the aforementioned effect of the non-linearities on the behaviour of vehicles at the stability limit.

4. Non-linear wheel/rail contact geometry parameters

4.1. Proposed parameters to characterise the wheel/rail contact

The equivalent conicity value for 3 mm wheelset amplitude is used today for characterisation of wheel/rail contact geometry [1,2]. The experience with use of this parameter in the railway community confirmed this parameter as useful information regarding the instability safety limits according to the standards for vehicle acceptance. The equivalent conicity, however, does not consider the non-linearity of wheel/rail contact.

As an approach to consider the non-linear wheel/rail contact geometry, a new description is proposed consisting of two parameters. The equivalent conicity as used today is extended with a second parameter related to the slope of the conicity function. This second parameter allows assessing the contact geometries with the same conicity level. Whereas the equivalent conicity provides a level (quantity) assessment regarding the instability, the proposed non-linearity parameter characterises the quality or the performance at this level. The equivalent conicity is related to the critical speed with respect to the instability safety limits, whereas the non-linearity parameter allows distinguishing if the critical speed can be expected to occur as a sudden flange-to-flange limit cycle or as a limit cycle with a small amplitude growing with increasing speed.

The proposed characterisation of non-linear wheel/rail contact geometry hereby consists of:

- Level parameter expressed by the equivalent conicity as used today, i.e. conicity value for the wheelset amplitude of 3 mm
- Non-linearity parameter λ_N related to the slope of the conicity function in the neighbourhood of the value for 3 mm wheelset amplitude.

The definition of the non-linearity parameter has been selected with the aim to allow an easy assessment applying the tools used today, so that the calculation of non-linearity parameter is possible with a small extension of the conicity assessment carried out by many railway operators and infrastructure companies today. According to the latest revision of UIC 518 [22], the conicity values for the wheelset amplitude of 2 mm and 4 mm should be assessed together with the conicity for 3 mm amplitude. Those three conicity values can certainly be proposed to calculate a non-linear parameter characterising an increase of the conicity function related to the increase of the wheelset amplitude by 1 mm for the range of wheelset amplitudes between 2 and 4 mm

$$\lambda_N = \frac{\lambda_4 - \lambda_2}{2} \qquad (16)$$

where λ_2 is the equivalent conicity for the wheelset amplitude of 2 mm and λ_4 the equivalent conicity for the wheelset amplitude of 4 mm.

The non-linear parameter definition (16) considers a sufficient lateral clearance between the wheelset and track before a flange contact occurs. To avoid a misrepresenting characterisation in case of a tight track gauge, an adaptation of the proposed parameter could be introduced analogue to the definition of the equivalent conicity in TSI High Speed [3] in regard to the maximum clearance wheelset/track. The Equation (16) will then read

$$\lambda_N = \frac{\lambda_{(y_\lambda+1)} - \lambda_{(y_\lambda-1)}}{2} \qquad (17)$$

with

$$
\begin{aligned}
&y_\lambda = 3\ \text{mm} && \textit{if} \quad (TG - FG) \geq 7\ \text{mm} \\
&y_\lambda = \frac{(TG - FG - 1)}{2} && \textit{if} \quad 5\ \text{mm} \leq (TG - FG) < 7\ \text{mm} \\
&y_\lambda = 2\ \text{mm} && \textit{if} \quad (TG - FG) < 5\ \text{mm}
\end{aligned}
\tag{18}
$$

where *TG* is the track gauge and *FG* the distance between the active faces of a wheelset, both in mm.
Alternative definitions of the proposed non-linearity parameter using other values of wheelset amplitude would be possible. Preliminary studies comparing different non-linearity parameter definitions did not show important influence on the results and further studies would be required for final judgement.

4.2. Assessment of wheel/rail contact geometry examples

The proposed characteristic parameters were compared and analysed on six examples of wheel/rail profile combinations with identical wheel and rail profiles on the left and right side. The investigated pairs were selected to represent three levels of equivalent conicity and at the same time two different contact non-linearities for each conicity level. The selected profiles consist of theoretical as well as worn profiles. They do not necessarily represent any typical or standard profiles, but were rather selected to reach completely different contact geometries and therefore different non-linearity effect while obtaining the selected conicity level for the nominal track gauge value of 1435 mm.
The following methods to calculate the equivalent conicity were applied:

- harmonic linearisation, elastic wheel/rail contact with a wheel load of 70 kN
- harmonic linearisation, rigid wheel/rail contact
- equivalent linearisation by application of Klingel formula according to UIC 519 [14], Appendix B
- linear regression of the Δr-function (difference of rolling radii) according to UIC 519 [14], Appendix C
- UK-method for a stochastic wheelset displacement with standard deviation of 1.25, 2.50 and 3.75 mm.

The calculations were carried out using the tools RSGEO [23] and VAMPIRE® [24] for a wheel diameter of 850 mm, wheelset back-to-back distance of 1360 mm and track gauge 1435 mm. A comparison of the equivalent conicity functions of the investigated wheelset/track combinations can be seen in Figure 5. Differences can be observed between the rigid and elastic wheel/rail contacts, namely in examples 1b, 2a and 3a. The rigid contact represents a limit case of the elastic contact for wheel load → 0. A large movement of the contact area occurs for very low wheel loads, whereas a lateral widening of the contact patch along with a rather continuous movement of the patch centre is present for heavy wheel loads as can be seen in Figure 6 on the combination of theoretical wheel and rail profiles 1b. This effect leads to differences in the Δr-function as shown in Figure 7a.
In some cases, large difference can be observed for wheelset amplitudes greater than 6 mm. This difference is related to the rotation of wheel profile about an axis longitudinal to the track (roll movement) due to the lateral wheelset displacement. This effect concluded to be negligible in the description of the Ayasse's method in [25] and also in the article by Gerlici and Lack [26] is neglected in UIC Code 519 [14] and EN 15302 [15]; the profiles are only shifted laterally and vertically. However, this rotation is considered when using simulation tools to calculate the equivalent conicity. This leads to a difference of the Δr-function; see the comparison for the profile combination 1b in Figure 7b as example. This difference is very small for wheel tread contact but not negligible in case of a flange contact. For the contact in the wheel tread area, however, the equivalent conicity functions possess similar shapes.

Surprisingly, the equivalent conicity values calculated using the UK-method are also comparable with other methods, in spite that this method uses a stochastic instead of a periodic wheelset movement.

A comparison of the equivalent conicity values for 3 mm wheelset amplitude calculated using different methods is shown in Figure 8. In spite of similar shape of conicity functions, the equivalent conicity deviates dependent on the calculation method. Unsurprisingly, the largest differences occur between the conicity values calculated by the UK-method for the wheelset displacement standard deviation of 2.5 mm and other methods calculated for a periodic wheelset displacement with an amplitude of 3 mm. However, the results calculated under the assumption of a periodic wheelset displacement deviate as well. In the investigated examples, the difference reaches up to 0.14 which is more than 30% of the conicity value. A comparison of both methods described in UIC 519 and EN 15302 results in differences 0.02-0.03 except for case 3b where a difference of 0.085 occurs. Such a difference is higher than the value 0.05 which is specified in EN 14363 as a maximum conicity increase to avoid new tests if the vehicle has been tested with new instead of worn wheel profiles. Even if the two methods described in UIC 519 and EN 15302 are closer to each other we have to keep in mind the simplifications applied in those methods (rigid contact, neglected roll movement of wheel profiles during the wheelset displacement). The equivalent conicity value calculated using the methods described in UIC 519 and EN 15302 is therefore less representative for the vehicle's behaviour than an equivalent conicity value calculated using the elastic wheel/rail contact and considering the complete movements of wheel profiles including the rotation about an axis longitudinal to the track.

As conclusion from this comparison we can state that the equivalent conicity value for 3 mm wheelset amplitude as used for the specification of wheel/rail contact conditions during the vehicle acceptance is only an indication. It is only comparable as long as the same method and calculation tool is used. The limit for the conicity increase due to wheel wear as used today in EN 14363 is too small; it is even smaller than the deviation which can occur when assessing the same pair wheelset/track by different conicity calculation methods used in the railway community.

The proposed characteristic parameters of the investigated combinations wheelset/track are shown in Figure 9. This description consists of the non-linearity parameter according to (16) together with the equivalent conicity for 3 mm wheelset amplitude calculated by harmonic linearisation using the elastic contact. As can be seen in Figure 9, the analysed pairs wheelset/track represent three conicity levels, whereby there is one pair with a negative and one with a positive non-linearity parameter for each conicity level.

4.3. Characteristic parameters and vehicle dynamic behaviour

The influence of wheel/rail contact non-linearity on the bifurcation diagram of a non-linear double-decker coach model and the investigated wheelset/track pairs is illustrated in Figure 10. Large wheelset amplitudes above approximately 5 mm are reached at similar speeds for both wheel/rail contact geometries with the same level parameter (equivalent conicity λ), so that the instability safety limits will be achieved at similar speeds for the same conicity. However, the shape of the bifurcation diagrams and the appearance of a limit cycle vary significantly due to different non-linearity parameters λ_N.

The trends of bifurcation diagram alterations due to changes of the wheel/rail contact characteristic parameters are shown in Figure 11. An increase of the level parameter (equivalent conicity) leads to a decrease of speed at which the instability safety limits will be exceeded. An increasing non-linearity parameter promotes the bifurcation diagram with the subcritical Hopf bifurcation. A sudden occurrence of a limit cycle with large amplitude can be expected at the stability limit. The running dynamics of vehicles is certainly dependent on all non-linearities of the system vehicle/track; hence, the bifurcation diagrams can significantly differ for other vehicles. However, similar trends in relation to the proposed wheel/rail characteristic parameters observed by the author in simulations of other vehicles confirm the presented tendencies to be rather general.

The effect of characteristic parameters on the critical speed of the investigated double-decker coach can be seen in Figure 12. Considering the non-linear critical speed defined by a presence of a limit cycle (Figure 12a), large differences can be observed. The influence of the non-linearity parameter on the critical speed is even higher than the effect of the equivalent conicity. When considering the critical speed defined by a presence of the limit cycle with the same wheelset amplitude (Figure 12b), the differences reduces and the critical speed is influenced mainly by the conicity as known from on-track measurements. Hence, the differences in critical speed calculations [27] or differences between the measurements and simulations could possibly be explained using the proposed non-linearity parameter.

Figure 13 presents the relationship between the wheel/rail characteristic parameters and the simulations of vehicle run on a straight track with two sets of measured irregularities. The results show higher values of lateral bogie accelerations and of the sum of guiding forces in simulations with the wheel/rail contact geometry "Type B", i.e. with the negative non-linearity parameter, as also confirmed in [28]. The rms values of the lateral acceleration on a bogie frame and the sum of guiding forces are decreasing with decreasing conicity, but also with increasing non-linearity parameter, whereby the effect of the non-linearity parameter is even more important than the effect of the conicity. The relationship between the wheel/rail characteristic parameters and the maximum values of accelerations and wheel/rail forces show less obvious, but still similar tendency.

The rms and peak values of the bogie frame accelerations as well as the wheel/rail forces are similar although the track A is of better quality compared with track B as documented in Figure 14. The standard deviation of lateral irregularities achieves 1.1 mm at track A compared to 3.3 mm at track B, and the peak value of 5.0 mm compared to 11.7 mm. This comparison demonstrates that the wheel/rail contact geometry can have a dominating effect on the lateral vehicle dynamic behaviour. The proposed wheel/rail contact geometry characterisation can thus enhance the assessment of the running dynamics conditions.

5. Summary and outlook

This article deals with the characterisation of wheel/rail contact geometry for measured or theoretical wheel and rail profile assessments during the vehicle testing or for the specification of multi-body simulations. First, the traditional characterisation of wheel/rail contact geometry using the quasi-linearisation and its limitations are presented. Then, the effect of the non-linearity of the contact geometry wheelset/track on the behaviour of vehicles at the stability limit is shown.

A new description characterising the wheel/rail contact geometry by two parameters is presented. The first parameter allows assessment of the vehicle performance regarding the instability safety limit as specified in EN 14363 [1]. The newly introduced, second parameter allows assessment of the expected behaviour at the stability limit: either a sudden flange-to-flange limit cycle or a limit cycle with a small amplitude growing with increasing speed. This parameter also shows the sensitivity of the vehicle to the lateral excitation by track irregularities. The proposed wheel/rail contact geometry description is compared on six examples of contact geometries wheelset/track with three different levels of equivalent conicity. The relationship between the characteristic parameters, bifurcation at the stability limit, critical speed and dynamic behaviour of a vehicle on measured track irregularities is shown.

The presented definition of characteristic parameters allows an improved but still comprehensive description of non-linear wheel/rail contact geometry. It could contribute to better understanding of railway vehicle behaviour for different wheel/rail contact conditions during testing, more exact assessment of wheel conditions (check for need for wheel maintenance), rail profiles (check for need for rail maintenance) and to more detailed specification of the wheel/rail contact geometry in multi-body simulations.

This article is only the first step on the way to improved generalised characterisation of wheel/rail contact geometry. Further assessments of measured data, extensive simulations using measured wheel/rail contact geometries and additional analyses of on-track tests are required to confirm the observed relationships, to assess the applicability of the proposed characterisation or to identify better suited parameters. This effort would provide the knowledge necessary to establish an improved but still simple characterisation of wheel/rail contact geometry. A closer relationship of these characteristic parameters to running dynamics of non-linear railway vehicle systems would increase the value of the wheel/rail contact geometry measurements and assessments.

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Figures:

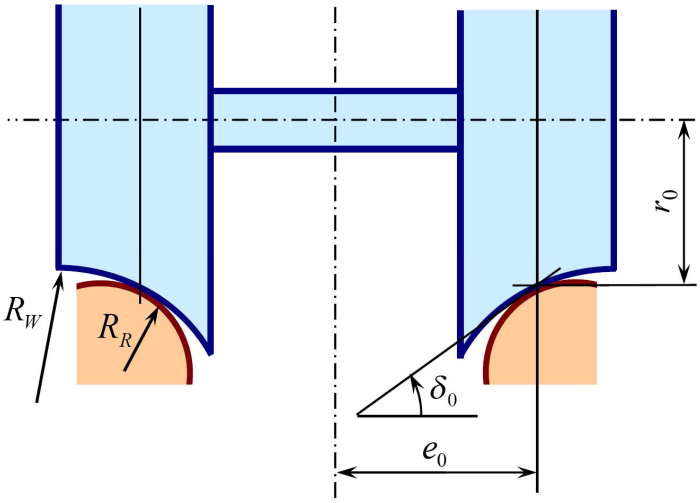
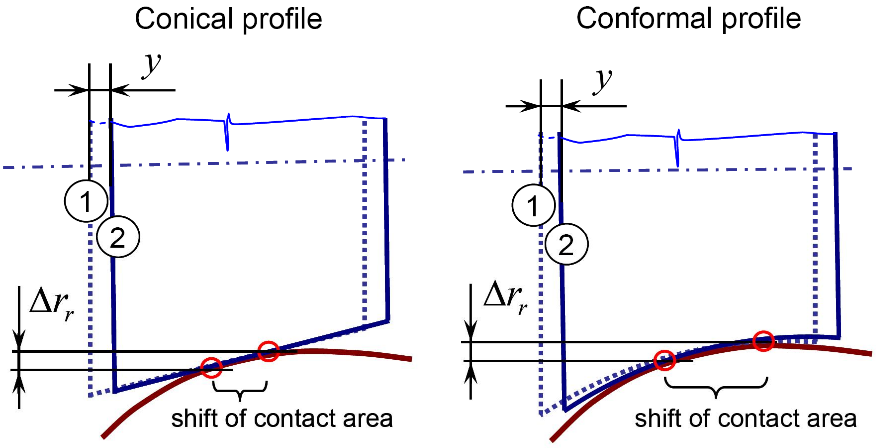


Figure 1. Linearised model wheelset/track.


Conical profile

Conformal profile

Figure 2. Lateral shift of the contact area across the conical and conformal wheel profiles due to wheelset displacement leading to the same rolling radii difference.

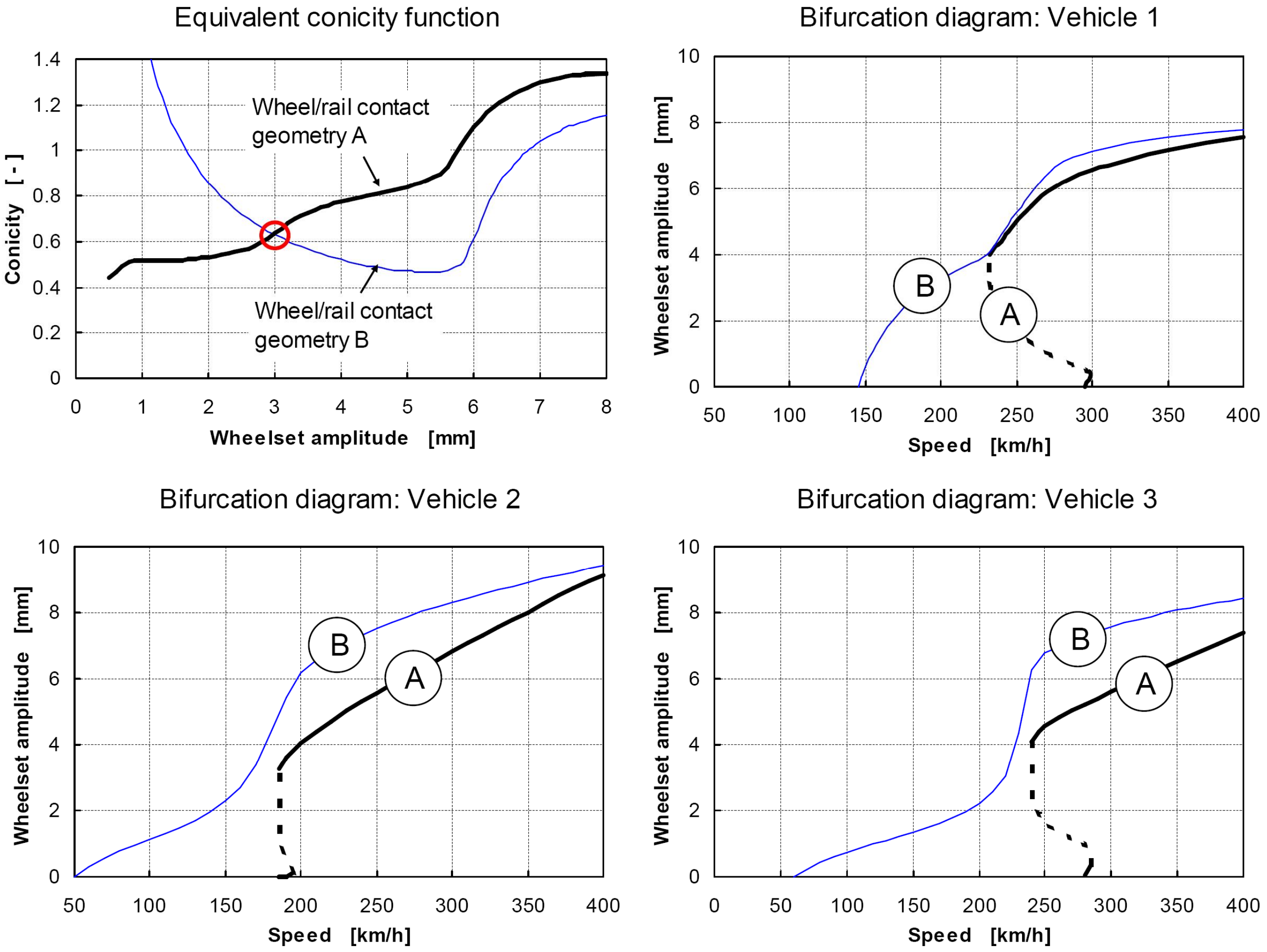


Figure 3. Effect of wheel/rail contact non-linearity on the bifurcation diagram. Vehicle 1: Articulated EMU, Vehicle 2: Double-decker coach without yaw dampers, Vehicle 3: Double-decker coach with yaw dampers.

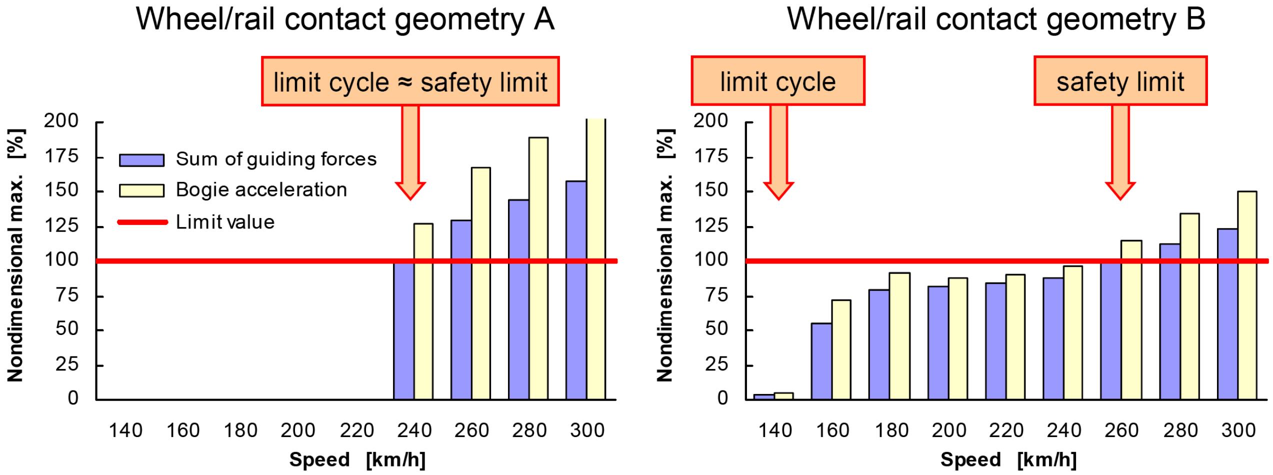


Figure 4. Relationship between the non-linear critical speeds identified by a presence of a limit cycle and by an exceedance of the instability safety limits according to EN 14363 after an excitation by a single lateral disturbance (Vehicle: Articulated EMU).

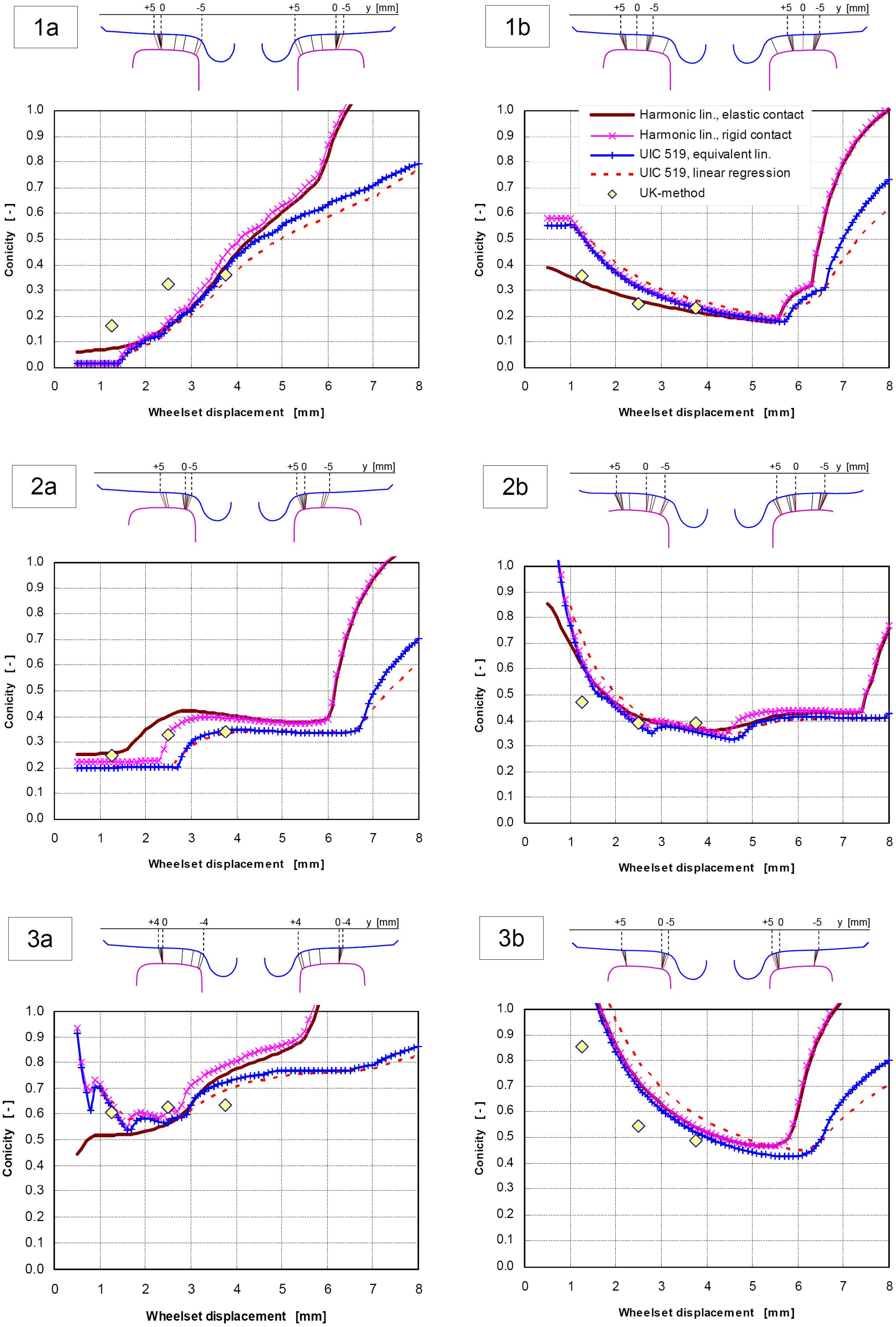


Figure 5. Conicity functions of the investigated wheel/rail contact geometry examples.

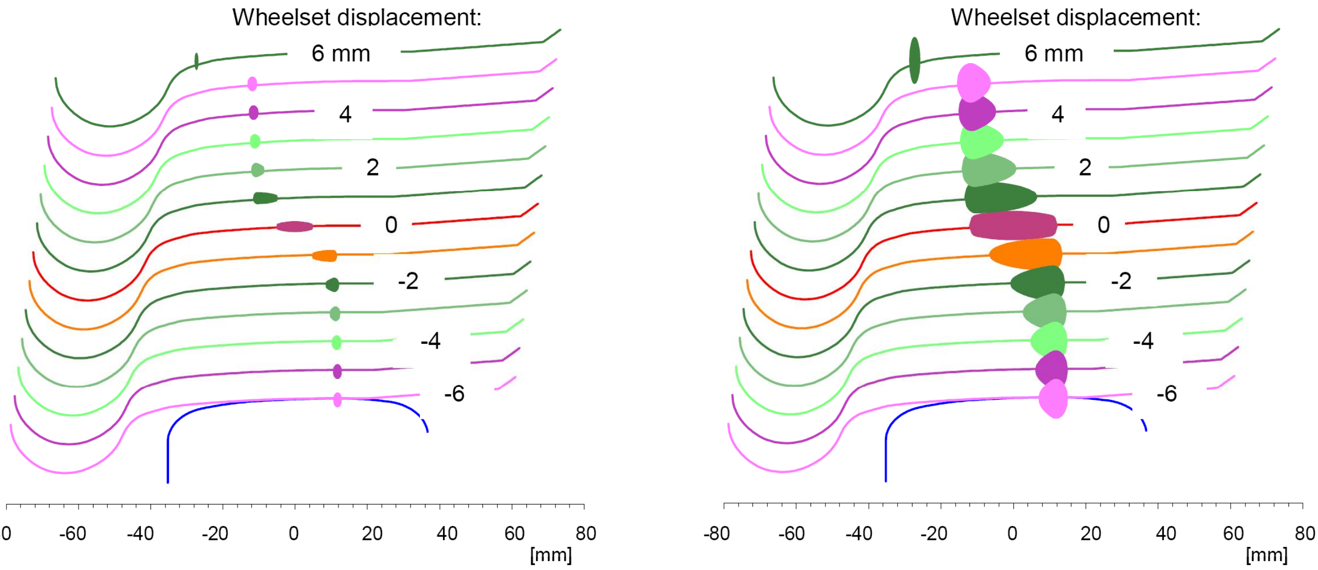
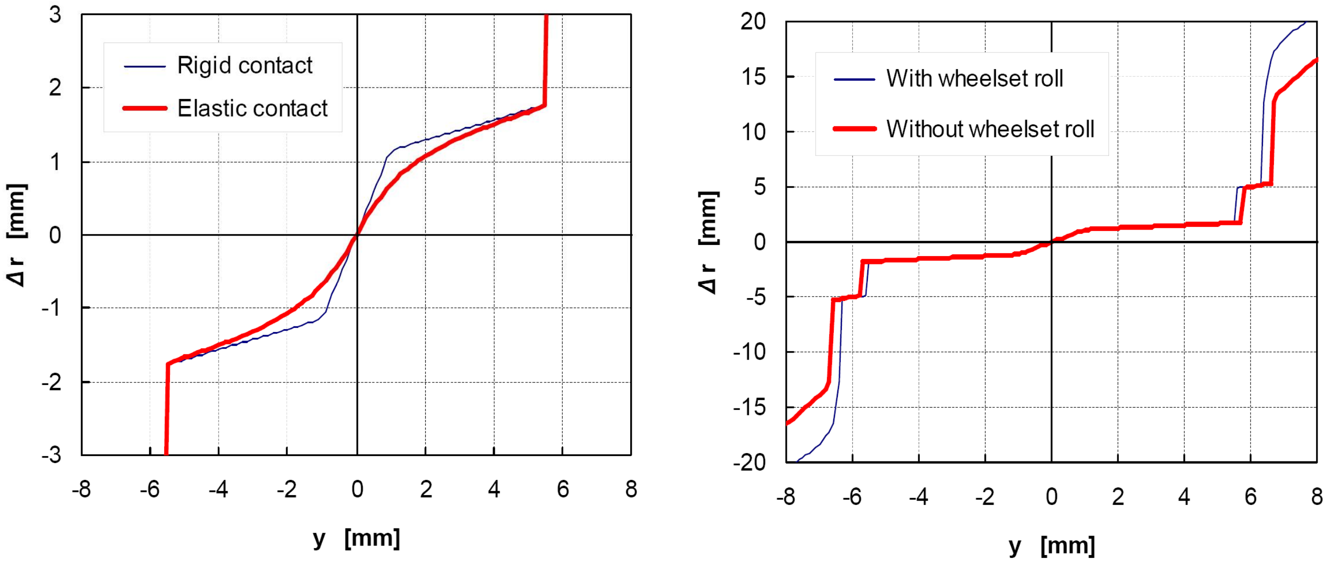


Figure 6. Effect of wheel load on the size and the lateral shift of the contact patches across the profiles when using an elastic contact model.



(a) (b)

Figure 7. Effect of contact elasticity (a) and roll movement of wheel profiles (b) on the rolling radius difference function on the example of the profile combination 1b from Figure 5.

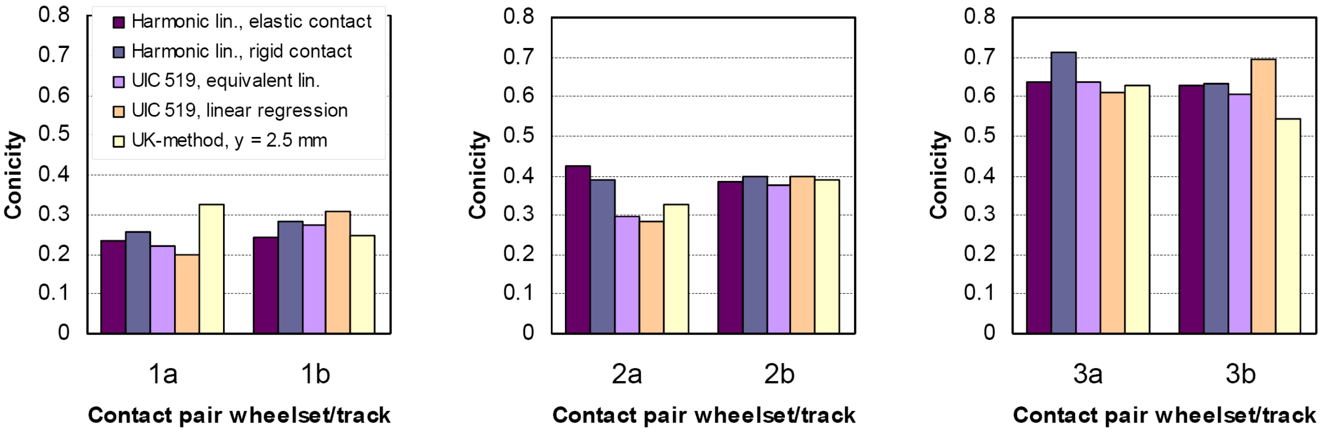


Figure 8. Equivalent conicity comparison of the investigated wheel/rail contact geometry examples.

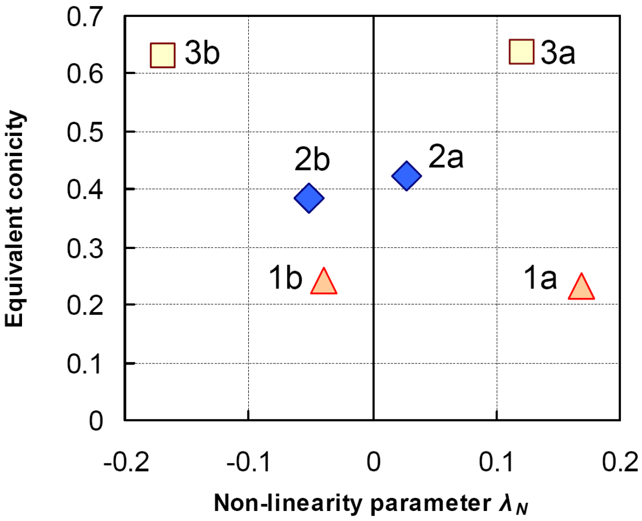


Figure 9. Characteristic parameters of the investigated wheel/rail contact geometry examples.

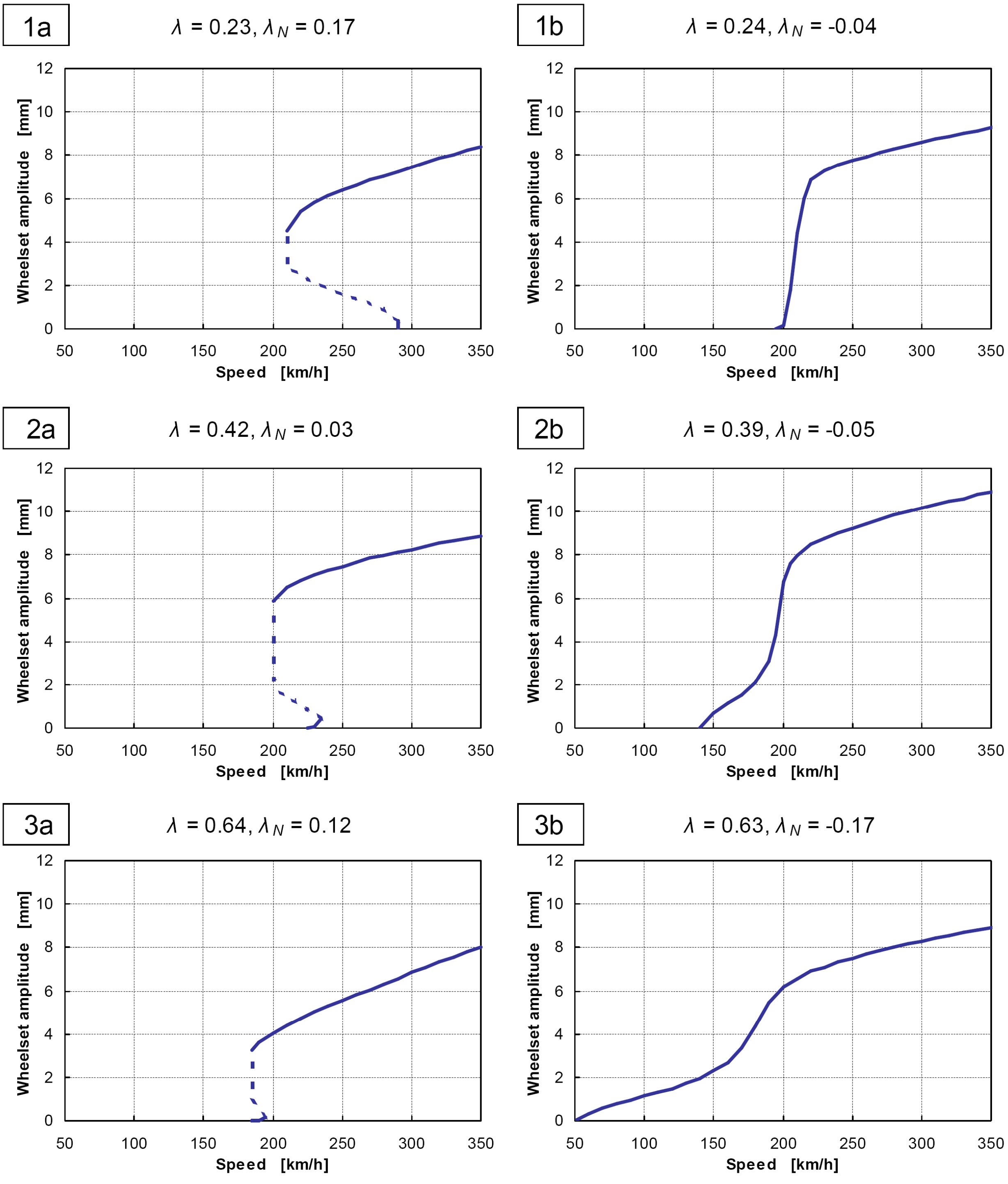


Figure 10. Bifurcation diagrams of a double-decker coach without yaw dampers and the investigated wheel/rail contact geometries.

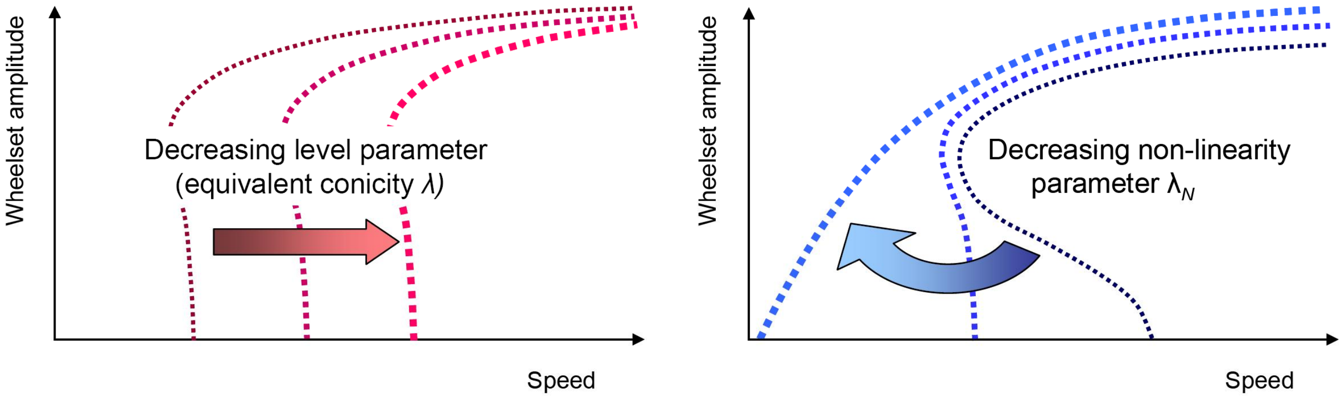


Figure 11. Influence of wheel/rail contact geometry on the bifurcation diagram: Trends due to the variation of the level and non-linearity parameters.

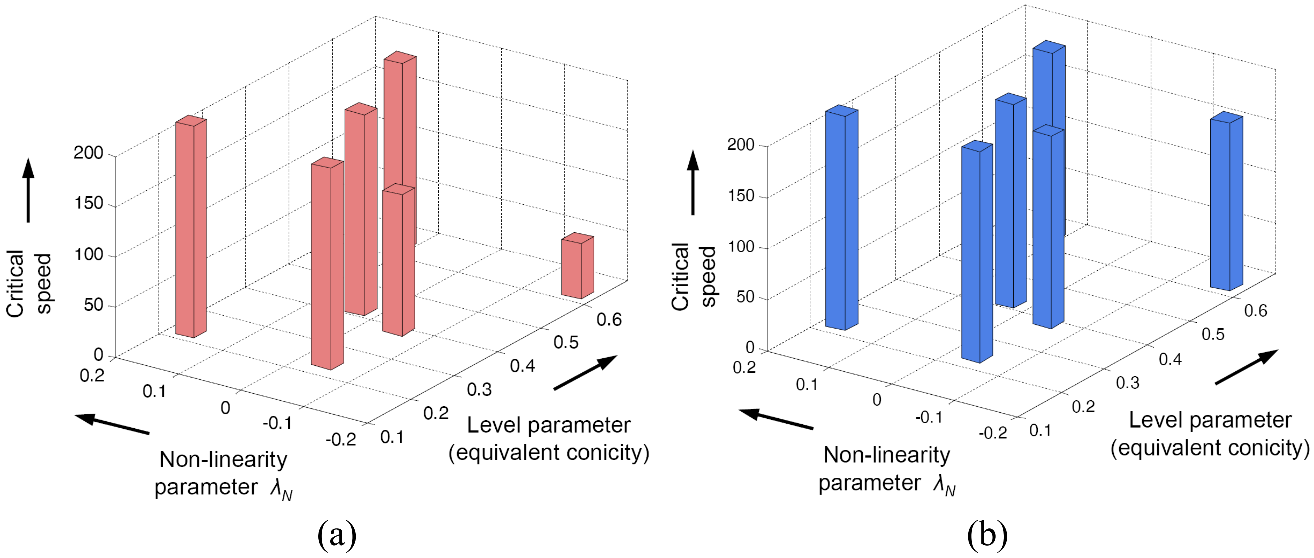


Figure 12. Effect of non-linearity parameter on the critical speed:
a) Critical speed defined as the lowest speed at which a limit cycle occurs.
b) Critical speed defined by the presence of a limit cycle with 3 mm wheelset amplitude.

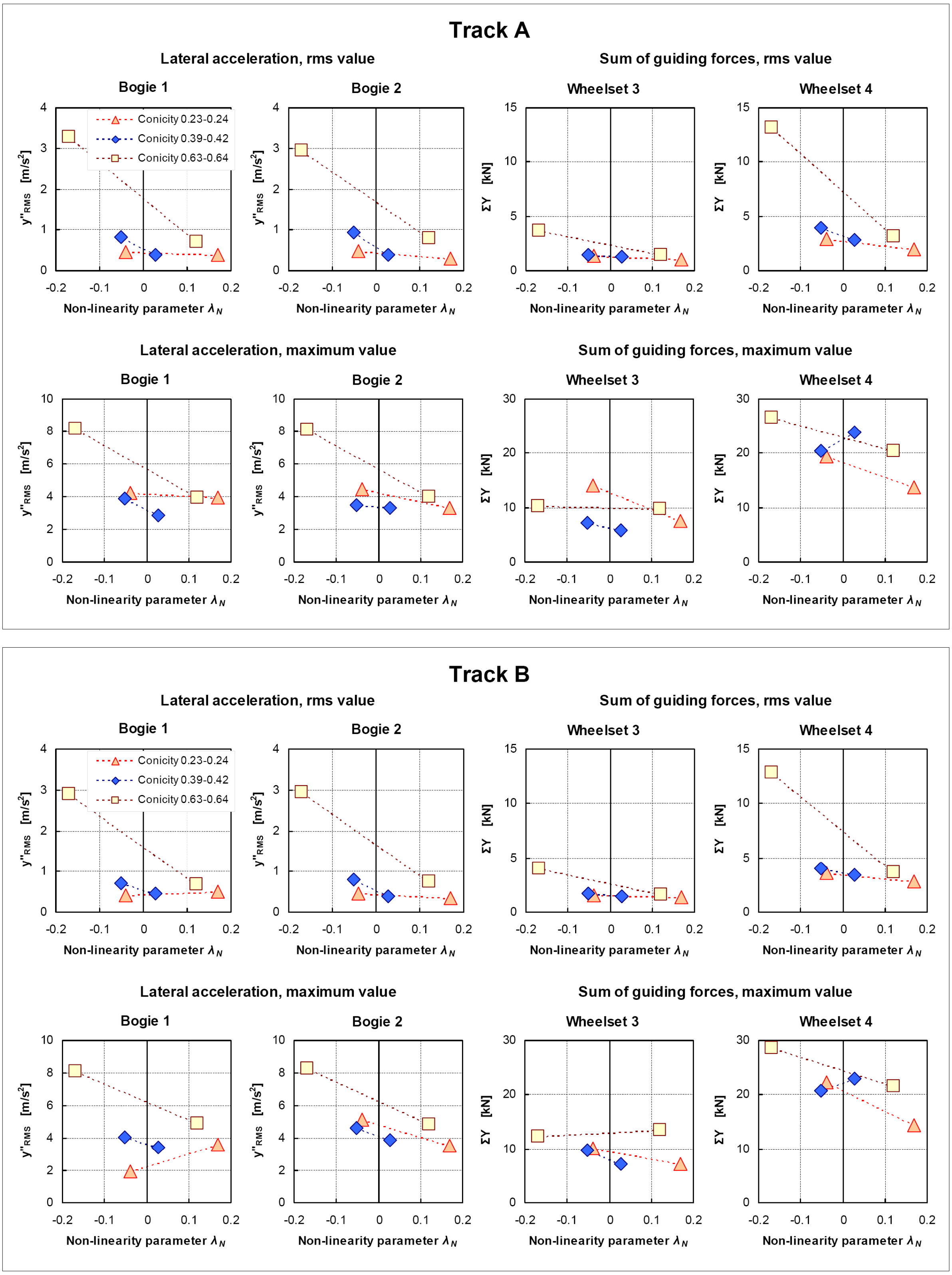


Figure 13. Relationship between the proposed wheel/rail characteristic parameters and the vehicle dynamic behaviour on measured track irregularities. Simulation results for a double-decker coach with yaw dampers at speed of 160 km/h.

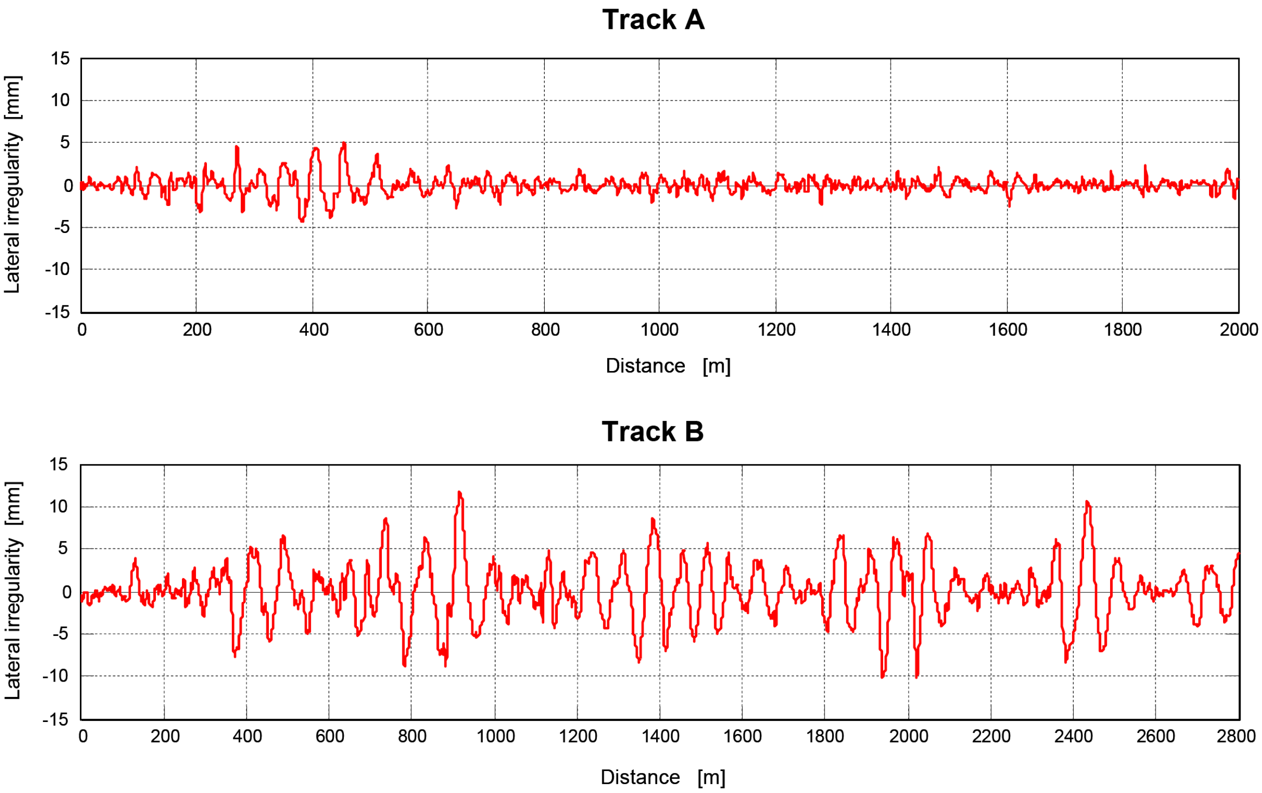


Figure 14. Comparison of lateral track irregularities used in the simulations presented in Figure 13.